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CONTENTS

PAGE

EDITORIAL—Napier Shaw Centenary 1

REVIEWS OF MODERN METEOROLOGY—11. The physical state of the upper atmosphere—K. Weekes 2

F. H. BUSHBY and M. K. HINDS—Computation of tendencies and vertical motion with a two-parameter model of the atmosphere 16

T. W. R. EAST and J. S. MARSHALL—Turbulence in clouds as a factor in precipitation 26

W. A. MORDY and L. E. EBER—Observations of rainfall from warm clouds (Plate I) 48

A. R. CURTIS and R. M. GOODY—Spectral line shape and its effect on atmospheric transmissions 58

R. S. SCORER—The nature of convection as revealed by soaring birds and dragonflies 68

G. REYNOLDS—Rainfall in the Isle of Man 78

Shorter Contributions—

A. C. BEST—Dimensionless equations for the growth of a drop of salt solution .. 89

T. H. ELLISON—On the correlation of vectors 93

A. R. MEETHAM—Natural removal of atmospheric pollution during fog 96

M. A. MAHROUS—Drop sizes in sea mists 99

Correspondence—

J. PEPPER—Sunshine in the southern Sudan 102

A. F. BUNKER ; F. H. LUDLAM and R. S. SCORER—The stable sub-cloud layer and convective updraughts 102

K. H. SOLIMAN—Rainfall over Egypt 104

L. ALPERT—Showers from cumulus congestus clouds 105

W. E. SAUNDERS—Night cooling under clear skies 105

Discussions 106

G. K. BATCHELOR—The conditions for dynamical similarity of motions of a frictionless perfect-gas atmosphere (*Q.J.*, **79**, p. 224); F. A. BERSON—A quantitative analysis of the evolution of large-scale flow with regard to the effect of eddy motion (*Q.J.*, **79**, p. 210); P. D. THOMPSON—On the theory of large-scale disturbances in a two-dimensional baroclinic equivalent of the atmosphere (*Q.J.*, **79**, p. 51); G. W. HURST—(a) The profile of a jet stream observed 18 January 1952 (*Q.J.*, **78**, p. 613), (b) The profile of a jet stream observed 1 September 1952 (*Q.J.*, **79**, p. 407); D. H. JOHNSON—A further study of the upper westerlies; the structure of the wind field in the eastern North Atlantic and western Europe in January 1950 (*Q.J.*, **79**, p. 402); R. MURRAY and S. M. DANIELS—Transverse flow at entrance and exit to jet streams (*Q.J.*, **79**, p. 236).

Proceedings at meetings of the Society 114

Reviews 117

Deep-sea Research, **1**, No. 1 (T.W.W.); *Advances in Geophysics*, I, Ed. H. E. Landsberg (P.A.S.); *Modern developments in fluid dynamics: High speed flow*, Ed., L. Howarth (C.J.T.); *The scientific work of René Descartes* (1596-1650), J. F. Scott (E.G.); *Waves and Tides*, R. C. H. Russell and D. H. Macmillan (F.U.); *The skin of the earth*, A. Austin Miller (G.M.H.); *Publication of the Institute for Weather and Climate Research of the Norwegian Academy* (G.A.B.) ; *Emploi de la hauteur barométrique comme coordonnée verticale*, O. Godart (G.A.B.); German Climatological Atlases (G.A.B.); *Weather inference for beginners*, D. J. Holland (R.M.P.); *Atmospheric electricity*, B. F. J. Schonland (E.T.P.); *Fundamentals of thermometry* and *Practical thermometry*, J. A. Hall (R.F.); *Annuaire de l'Observatoire Aérologique à Belgrade, 1951* (G.A.B.); *Values of some physical functions and constants used in meteorology. Definitions and specifications of water vapour in the atmosphere* (A.W.B.); *Faster than thought*, Ed., B. V. Bowden (E.N.M.); *Le Pioggie della Libia*, A. Fantoli (A.F.J.); *The Antarctic today*, New Zealand Antarctic Society, Ed. F. A. Simpson (H.H.L.); *Thunderstorm Electricity*, Ed., H. R. Byers (F.J.S.)

Obituary Notices 127

Corrigenda 130

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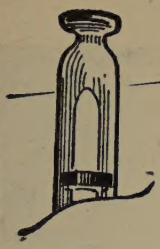

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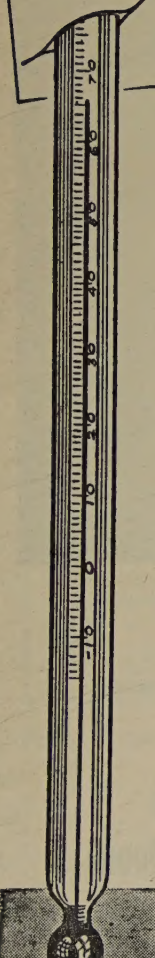

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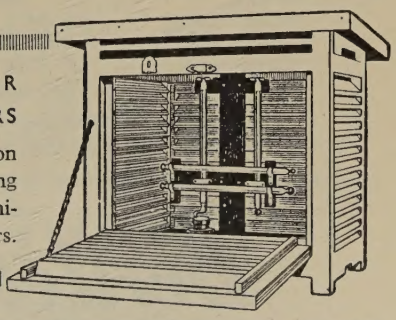

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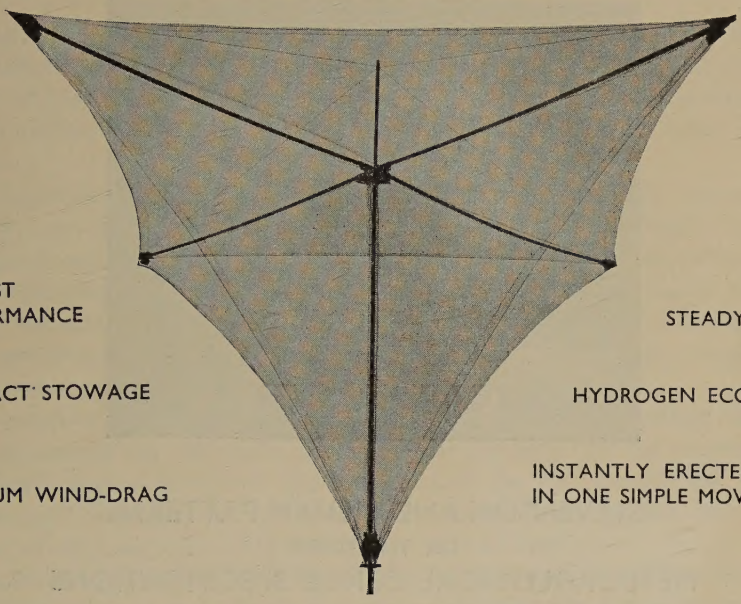

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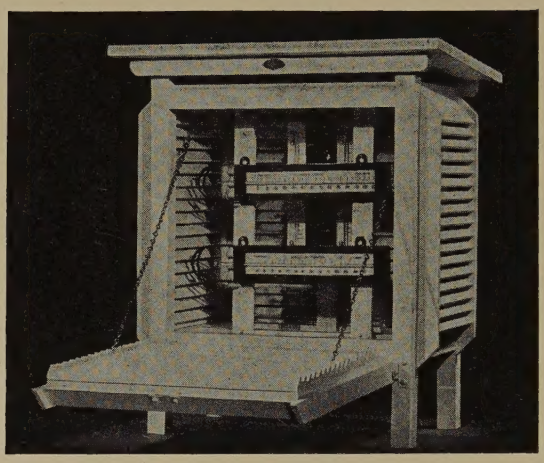

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EDITORIAL

The fourth of March, 1954, will be the centenary of the birth of Sir Napier Shaw. His life extended into its tenth decade, spanning an era of scientific as of political revolution. With several of his near contemporaries, for longevity seems almost a characteristic of his scientific generation, he was a link with and an embodiment of a more spacious and gracious age.

His remarkable career went through three main phases. He first established a reputation as a successful college tutor and university teacher of physics, an occupation which he abandoned, only at the turn of the century, to become associated for the first time in an administrative capacity with official meteorology. Finally, at the age of sixty-six, he became the first Professor of Meteorology at Imperial College and conceived the ambitious plan of a four-volume *Manual of Meteorology*, a task which was completed eleven years later.

The breadth of his interests and the depth of his scientific insight, the ability to grasp and to express the essentials of a problem, his work for the organization of an adequate Meteorological Service in this country and also for International Meteorology, and his personal character, his qualities of kindliness and courtesy, won the esteem and affection of all those who worked with him or knew him.

As a result of an appeal to the Fellowship and to Scientific Instrument Manufacturers, a memorial fund has been raised by the Society to commemorate the unique position which Shaw achieved in British Meteorology. Council have decided that this fund shall be employed in the regular award of a substantial prize for an original monograph on a subject which will be specified well beforehand. The prize will be awarded as the result of an open competition.

The detailed announcement of the first competition for the Napier Shaw Prize will be made very shortly.

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The physical state of the upper atmosphere

By K. WEEKES
Cavendish Laboratory, Cambridge

1. Introduction

Routine measurements of the temperature and pressure of the atmosphere are now made to heights of rather more than 20 km. These measurements give reliable data about 95 per cent of the mass of the atmosphere. The remaining 5 per cent is relatively inaccessible, lying above 20 km and extending to several hundred kilometres above the ground; for the purpose of this survey we shall define this 5 per cent of the atmosphere as the upper atmosphere.

In spite of its small bulk the upper atmosphere is of considerable scientific interest particularly because of the several natural phenomena which have their origin within it. Thus ozone lies mainly between 15 and 40 km, the aurorae occur in the neighbourhood of 100 km and the region above about 80 km is sufficiently ionized to play an overwhelmingly important role in the propagation of radio waves to great distances on the earth.

Until the use of high-altitude rockets for scientific research became possible, in about 1946, the only way in which information could be obtained about the high atmosphere was by the study of these naturally occurring phenomena. The general procedure was necessarily to formulate a theory of the phenomenon in terms of known physical laws and, by comparing the predictions of the theory with the observations, to verify that the theory was essentially satisfactory and to deduce from its adjustable parameters some properties of the atmosphere. By these methods a more or less consistent picture of the upper atmosphere was built up in the last 30 years or so. It must be emphasized that the approach was very indirect and many of the theories very much idealized; often the detailed agreement between a theory and the observations was not good.

The advent of the rocket has considerably altered the situation since it becomes possible, albeit with some difficulty, to make direct observations of many of the static properties of the atmosphere. The information available from these direct experiments makes it possible to check the various theories of upper atmospheric phenomena more accurately and hence to strengthen or reject the deductions made through them. The rocket does not, at the present stage, render the other methods of investigation obsolete, since it is not possible to use rockets either very frequently or at all points of the earth's surface; thus the rocket may be thought of as providing data on which satisfactory interpretations of other phenomena may be based and these other phenomena may then be used to provide ' synoptic ' data about the high atmosphere.

The quantity which is very frequently ' measured ' by the indirect methods is the local atmospheric ' scale height,' $H = \mathbf{R}T/Mg$ and we shall have occasion to discuss various direct or indirect measurements of H. In general in the literature the ' temperature ' T is quoted; this is obtained from the scale height by assuming a value for the mean molecular weight M. For this reason we shall discuss first the composition of the atmosphere at various levels, mainly as regards the important constituents N_2 and O_2; this will give some indications of the value of M to be chosen in converting scale heights to temperatures. After the properties of the ' static ' atmosphere have been discussed, we shall review the evidence for movement in the high atmosphere and the possible complications which may be expected in the upper levels of the ionosphere as a result of the interaction of the ionized gas and the permanent magnetic field of the earth.

2. The composition of the atmosphere

It is well known that the composition of tropospheric air is essentially independent of the location of the observing point, only water vapour and carbon dioxide being important, variable constituents. Analyses of air samples from moderate heights in the stratosphere show essentially the same composition (Paneth 1937).

At higher levels the composition may be expected to alter as the result of diffusive separation of the different gases tending to make each vary according to its own scale height and also as a result of the photochemical action of solar ultra-violet radiation.

(a) *Diffusive separation*

In the troposphere the tendency towards diffusive separation is opposed by the mixing action of turbulence and wind systems; where the temperature gradient becomes zero or positive we may expect the atmosphere to be relatively stable and diffusive separation might set in. At the pressures found in the lower stratosphere, up to 40 km, the time needed to achieve diffusive separation is very long and so occasional disturbance would be adequate to upset the diffusion. As the pressure is decreased the time for diffusion decreases and above about 100 km is only a matter of days; thus we may expect to find diffusive separation more important at the higher levels of the stratosphere. Even at these levels, as we shall see later, there is evidence for very considerable winds which will oppose diffusion. Between 1932 and 1937 attempts were made by Paneth, using He, and Regener, using O_2 to detect any diffusive separation in the stratosphere below 30 km. Relatively small samples of air were collected in glass vessels carried up by balloons. These samples showed on analysis that there was a deficiency of O_2 and an increase of He in the samples; the two sets of results were not consistent, the decrease of O_2 with height being apparently more rapid than the increase in He, a very unlikely result (see Paneth 1937).

It is always extremely difficult to prevent loss of O_2 from a closed vessel and consequently it is not a satisfactory gas for this type of experiment in spite of the ease with which the analysis can be carried out. Further, as Paneth has pointed out, He is very liable to be absorbed in glass and the amount found in a glass vessel may be richer or poorer than in normal air whatever the origin of the sample. Thus for a satisfactory experiment it is necessary to use better collecting vessels and preferably to collect samples from a greater height.

A number of samples have been collected in steel vessels, opened and closed at various heights during rocket flights (Loh *et al.* 1952). These samples have been analysed in detail for the proportion of He, Ne, and A relative to N_2; the oxygen is again quite unreliable and several samples contained effectively no O_2.

These samples collected between 60 km and 95 km have shown that the lighter gases are enriched and the heavier depleted relative to N_2, as might be expected if diffusive separation were taking place (Chackett *et al.* 1951). Test measurements were made with ground-level air sampled under conditions as near that of the rocket sampling as possible. Four such samples gave rather inconsistent results, two showed no change and two changes of the same nature as apparent diffusive separation. The results of the various analyses are shown in Table 1 (ref. G. 4, i.e., General References, 4).

These results are not entirely satisfactory, the effects in the different gases are not consistent and the separation, particularly in the case of argon, seems to tend to a limiting value at high altitudes. At the high altitudes the amount of gas in the samples was much less than would be expected. Martin has applied Knudsen's formula for the streaming of a gas down a tube and has found that, under the sampling conditions, some apparent

TABLE 1. PRESSURE AT ROCKET NOSE, PRESSURE OF SAMPLE AND AMOUNTS OF HELIUM, NEON AND ARGON, RELATIVE TO THEIR AMOUNTS IN GROUND-LEVEL AIR, FOUND IN SAMPLES FROM VARIOUS HEIGHTS.

Height (km)	Pitot pressure (mm Hg)	Bottle pressure (mm Hg)	He	Ne	A
57·4	0·78	0·70	0·998	1·005	1·001
59·7	0·49	0·42	1·035	1·008	0·996
62·3	1·18	—	1·133	1·040	0·962
65·6	0·39	0·043	1·44	1·08	0·930
68·3	0·25	0·022	2·02	1·18	0·89
70·6	0·165	0·021	2·41	1·20	0·85
69·8	0·43	0·021	1·570	1·232	0·90
88·4	0·018	0·0016	2·943	1·395	0·82
93·3	0·0073	0·00034			0·82
104·3	0·0011	—			

separation would be observed and the separation ratio would tend to a limiting value equal to the square root of the mass ratio. These predictions appear to fit the experimental data surprisingly well (E. Martin, ref. G. 4).

Preliminary results of a mass spectrographic study of the upper atmospheric gases between 100-130 km indicate that the A/N_2 ratio is very close to that at ground level (Townsend and Meadows, ref. G. 4).

Thus, at the present stage of the investigations, it seems unlikely that there is appreciable diffusive separation below about 100 km.

(b) *Photochemical action* (see Bates, ref. G. 3)

No solar radiation with wavelength less than about 2,900 Å reaches the 20-km level, so that all this ultra-violet radiation from the sun is absorbed in the upper atmosphere where it produces dissociation and ionization of the normal atmospheric gases and various photochemical reactions take place at different heights. Evidence of these effects may be had largely from spectrographic studies of the air glow and of the auroral radiation, and also from absorption spectra.

The most important photochemical changes are concerned with the oxygen. Absorption spectra show that there is an appreciable quantity of ozone in the atmosphere, while the emission spectra show very largely the presence of atomic oxygen. The ozone is produced at a height between 20 km and 50 km where there is a relatively high gas density and weak dissociation of molecular oxygen. At heights near 100 km, strong absorption of radiation of wavelength between 1,300 and 1,750 Å leads to the dissociation of molecular oxygen which recombines by way of three-body collision so that the efficiency of the recombination process decreases rapidly with height.

Many calculations of the photochemical equilibrium of oxygen and ozone have been made; the results indicate a distribution of ozone in fair agreement with that measured. Similarly calculations of the equilibrium of atomic and molecular oxygen indicate a transition at about 100 km. Measurement by rockets of the intensity of solar radiation near 1,450 Å, at the peak of the molecular oxygen absorption-band, shows a rapid increase between 90 km and 140 km, suggesting that the transition occurs between these heights (Friedman, Lichtman and Byram 1951). Exact calculations of the photochemical processes are impossible owing to lack of data on the various reaction rates. It is possible to estimate the times needed to establish photochemical equilibrium in the various processes. Thus for ozone it is found that near 50 km the time for reaching half value of the ozone concentration is a matter of minutes, while at 30 km where the available atomic oxygen

density is very small the time is of the order of 6 days. Similarly the time for half recombination of the atomic oxygen, in the absence of any dissociating process, is of the order 9 hr at 90 km and several months at 120 km. The times involved in these processes are in fact so long that it is a poor approximation to suppose the atmosphere to be static and the density of ozone, for example, at any place will vary with the air movements taking place. Evidence for such effects has been given by Dobson who found that there is appreciably more ozone over his apparatus during cyclonic weather conditions than during anticyclonic conditions. Detailed studies of these effects have been published, for example, by Langlo (1952) and Normand (1953).

In addition to atomic oxygen, the air-glow spectra show the presence of O_2, OH, Na and N_2; the auroral spectrum also shows the presence of N and N_2^+. The dissociation of N_2 by radiation is a rather inefficient process and is unlikely to produce marked effects; the time of half dissociation would be of the order of years. It is however possible that N_2 is dissociated indirectly by the reactions :

$$N_2 + h\nu \rightarrow N_2^+ + e$$
$$N_2^+ + e \rightarrow N' + N''$$

This type of recombination, of molecular ions and electrons, was suggested by Bates and Massey (1947) and was shown by Bates (1950) to be capable of considerable efficiency although precise calculations cannot be made in the present state of the theory. However, calculations of the rate of ionization, and hence of recombination, possibly suggest that this whole reaction would proceed rather slowly and so a calculation of the distribution of atomic nitrogen by simple static considerations is unlikely to be correct.

In calculating the atmospheric temperature from scale heights the problem of the dissociation of nitrogen is very important and at present is undecided. The presence of molecular nitrogen at all heights in the aurorae suggests that dissociation is nowhere complete and it seems probable that the proportion dissociated does not exceed some 25 per cent. The photochemical equilibrium of oxygen and water vapour and hence the distribution of OH in the atmosphere have been considered by Bates and Nicolet (1950) who have shown that water vapour would be entirely dissociated above 90 km into atomic oxygen and OH, while above 100 km the OH would be dissociated also. The presence of these heteronuclear molecules is of importance in the thermal equilibrium of the upper atmosphere since they have important bands in the infra-red and so may act as cooling agents in the high atmosphere.

The fact that, above 80 km, there is some ionization of the atmosphere is revealed by radio methods. This ionization is relatively unimportant in determining the scale height of the atmosphere, since it involves only a very small proportion of the atmospheric gases. It may be noted that if all the gas were ionized the scale height would be doubled but this is almost certainly not the case at any height at which measurements have so far been made.

One of the most important features of the photochemical changes that we have mentioned is the long time required to set up equilibrium, so that any calculation of the distribution must take account of molecular diffusion at heights above 100 km and of the possible air movements at all heights; it may be possible in time to make use of these impermanent gases to get some idea of the air movements which are taking place.

3. Temperature and pressure at different levels

The various indirect methods of investigation had by 1948 led to a consistent model of the temperature and pressure in the upper atmosphere; this model was summarized in the N.A.C.A. standard atmosphere and a very similar model adopted independently by Weekes

and Wilkes (1947). It is useful to summarize here the reasoning on which these models were based before discussing the modifications introduced by the more recent experiments with rockets.

(a) 0-30 *km*

In this range of height direct temperature measurements were available from sounding balloons; they showed the familiar lapse rate in the troposphere and the nearly constant temperature in the stratosphere.

(b) 30-60 *km*

Observations of the abnormal propagation of sound to great distances had led to the conclusion that the scale height must increase between 35 km and 55 km. Detailed measurements made it possible to deduce an approximate profile of scale height and from the velocity of the wave front of the sound wave across the ground a value of scale height at the top of the trajectory could be deduced. Very few experiments of this type, in which observations were taken successfully all round the source of sound, had been made and consequently it was difficult to separate effects due to an actual change in scale height and due to a positive wind gradient with height. The majority of measurements were interpreted as due to a temperature maximum near 50 km of about 350°K (see, for example, Whipple 1939). Reliable measurements taking account of the effects of winds indicated that the temperature maximum might not be greater than 290°K. These have been confirmed by the recent observations of Crary in Bermuda.

(c) 60-90 *km*

There were a number of indications that the scale height and the temperature decreased above 60 km. Radio measurements of the height of reflection of 18-km radio waves throughout the day indicated that the scale height was only 6 km (200°K) near 80 km (Budden, Ratcliffe and Wilkes 1939) but their deductions depended on the correctness of the theory of reflection of these waves. Pekeris (1937) showed that if the resonance theory of the semi-diurnal oscillation of barometric pressure was correct then the scale height must decrease above 60 km. Weekes and Wilkes (1947) showed that the average temperature between 70 km and 90 km must be about 200°K.

(d) 100-400 *km*

In this region of the atmosphere many data were deduced from ionospheric measurements. The observations of thickness and height variation of the layers were used in conjunction with Chapman's theory of layer formation to give values of scale height. The theory assumed the temperature constant, ionization by monochromatic light and a constant recombination coefficient between positive ions and electrons. For region E (at about 120 km) the predictions of the theory were obeyed to a first approximation and the scale height deduced was about 7 km to 11 km according to the method used. For region F_1 (180 km) the agreement with theory was about as good and gave $H = 30$ km. For region F_2 (230 km) the agreement was very poor and only thickness measurements were used to give $H = 80$ km ($T \simeq 3{,}000$°K); see Appleton (1939). Estimated recombination coefficients were roughly equal in E and F_1 and vastly different in F_2. Thus the basis of the estimates of temperature for F_2 region was very weak.

Further checks on the overall model may be obtained from radio measurements of the frequency of collisions, ν, of electrons with other particles.

Measurements gave $\nu = 10^6$ at 90 km, $2{\cdot}10^4$ at 120 km and 10^3 at 250 km. If these are interpreted as collisions with neutral particles, then the number density may be computed from the relation $\nu_{ea} = 1{\cdot}8 \times 10^{-8}\, n_a\, (T/300)^{\frac{1}{2}}$,in which n_a is the number density of neutral particles. The very small decrease of ν between 120 km and 250 km implies a very high temperature. However Cowling and others have shown that collisions with charged particles are also likely to be important and Cowling (1945) gave the expression for ν with ions as :

$$\nu_{ei} = 6{\cdot}06 \times 10^{-3}\, (n_+ + n_-)\, (300/T)^{3/2}$$

The measured value $\nu = 10^3$ at 250 km may be the result of collisions with ions, if the temperature $T \simeq 500°\text{K}$.

Thus the collisional-frequency evidence may be satisfied either by a high temperature 3,000°K above 150 km or by a comparatively low temperature ($\simeq$ 1,000°K) but not by any intermediate value; see, for example, Gerson (1951). At the present time no direct evidence has been given that the collisions are with charged particles; this ought to be possible since it implies that the observed collision frequency should be proportional to the electron density.

Confirmation of the general result up to 120 km was given by meteor observations, which again involve several assumptions.

Harang (1946) used observations of the intensity distribution of auroral luminosity to deduce scale heights which were in extremely good agreement with those deduced from radio data. Bates and Griffing (1953) have shown, however, that the theoretical basis of Harang's calculations was unsound.

More direct measurements of temperature were made by Vegard (1944, 1950) from a study of the intensity distribution of the rotation lines in an auroral band spectrum. From these measurements made over many years he deduced a value $T = 240°\text{K}$ for all levels from 100-160 km. More recent measurements by Petrie (1952) using the same method have given $T = 750°\text{K}$ at heights near 200 km.

A similar method used on the lines of OH by Meinel (1950) has given $T = 200°\text{K}$ and the height of emission as 80 km, confirming the general results deduced by indirect methods. Thus the deductions are reasonably consistent though it is possible to question the basis of almost any individual deduction; the value of temperature for 200 km is very uncertain from these arguments. The theory of a relatively high temperature is supported by the fact that in a sunlit aurora it is possible to observe resonance scattering by N_2^+ to heights of 1,000 km, but it may be significant that the emitters are charged particles and as we shall see later there are important electrodynamic forces in the high atmosphere.

An additional indication of a high temperature is given by the amount of helium in the atmosphere. The measured amount is about 10 per cent of the amount expected from the denudation of rocks and one way of resolving this discrepancy is to suppose that the helium has escaped from the high atmosphere, which implies a temperature of about 1,500°K.

4. Measurements with rockets

Many measurements have been made with rocket-borne instruments. While these measurements are direct there are a number of difficulties to be overcome; for example the rocket contains some absorbed gas which comes out as the ambient pressure is reduced; not all the fuel is burnt and so some fuel vapour is present round the rocket to contribute to the pressure.

The rocket measurements however have some internal checks since it is possible to measure the ambient pressure (up to 120 km) and the density. The scale height may be

deduced from the pressure gradient and from the ratio of pressure to density; if these independent methods agree it is probable that the atmospheric pressure and density are correctly measured. A full account of the rocket measurements of density and pressure has been given by Havens, Koll and La Gow (1952). The data up to 120 km appear to be wholly consistent and must be considered reliable.

Above 120 km, only density data are taken, as the pressure gauge becomes unreliable. The density is deduced from the excess pressure in a gauge when the rocket is moving through the air. The theory is based upon elementary kinetic theory arguments and agrees with the other methods at 100 km. By this means the density measurements have been extended to 220 km, at which height the density is deduced to be $(1{\cdot}0 \pm 0{\cdot}2) \times 10^{-7}$ g/m^3. The scale height at this level was about 30 km corresponding to a temperature of 450°K for a purely atomic (N and O) atmosphere or 750°K for N_2 and O (La Gow, ref. G. 4). The agreement between the rocket results and the indirect methods below 100 km is quite good. The rocket data indicate $T = 280$°K at 50 km and $T = 200$°K at 80 km.

It is important to note that to get a significantly higher density at 220 km it is necessary to suppose that the rocket data at 120 km are in error and at this level there appear to be independent checks in the rocket results.

A completely independent method of investigation has been used up to 80 km. In this method a series of grenades were ejected from the rocket at different heights, accurately known from the rocket trajectory, and the time of travel of sound measured with a series of microphones like that used for the study of anomalous sound propagation (Ference 1952). The time of travel may be used to give a value for the velocity of sound in a particular layer of the atmosphere and also the wind in the layer, since this affects the direction of arrival of the sound. The results of this method, used entirely at night, agree well with the other method of measurement.

Table 2 gives the average results from the rocket experiments; the number densities of particles are calculated by assuming that the mean molecular weight is 29 up to 90 km and decreases linearly to 24 at 120 km whence it remains constant.

At 220 km, the number density given by these results is only $2{\cdot}6 \times 10^{15}$ m^{-3}. At this density it might be expected that diffusion would be very rapid and the theory of F region becomes very obscure. The level at which solar radiation produces an important effect is determined by the absorption coefficient and is approximately the level at

TABLE 2. DENSITY ρ, PARTICLE DENSITY n, AND TEMPERATURE T AT VARIOUS HEIGHTS.

Height (km)	ρ (g/m^3)	n (/m^3)	T (°K)
0	$1{\cdot}3 \times 10^{3}$	$2{\cdot}73 \times 10^{25}$	291
20	1×10^{2}	$2{\cdot}1 \times 10^{24}$	213
40	4·07	$8{\cdot}6 \times 10^{22}$	263
60	0·40	$8{\cdot}4 \times 10^{21}$	264
70	0·13	$2{\cdot}73 \times 10^{21}$	228
80	$2{\cdot}7 \times 10^{-2}$	$5{\cdot}68 \times 10^{20}$	200
90	4×10^{-3}	$8{\cdot}6 \times 10^{19}$	212
100	$8{\cdot}3 \times 10^{-4}$	$1{\cdot}9 \times 10^{19}$	222
110	$2{\cdot}1 \times 10^{-4}$	$5{\cdot}0 \times 10^{18}$	240
120	5×10^{-5}	$1{\cdot}25 \times 10^{18}$	278
130	$1{\cdot}6 \times 10^{-5}$	$4{\cdot}15 \times 10^{17}$	318
140	$5{\cdot}9 \times 10^{-6}$	$1{\cdot}4 \times 10^{17}$	368
150	$2{\cdot}5 \times 10^{-6}$	$6{\cdot}5 \times 10^{16}$	418
160	$1{\cdot}0 \times 10^{-6}$	$2{\cdot}6 \times 10^{16}$	468

which the radiation is reduced to e^{-1} of its incident value. If the scale height be H, the number density cm^3 at this level is $n = 1/AH$ where A (cm^2) is the absorption cross section. Thus for $H = 30$ km and $n = 2{\cdot}10^9$, $A \simeq 10^{-16}$ which is much larger than would be expected for any ionizing process. Thus it is difficult to see how F_2 region could be formed at such a height and further Ferraro (1945), and Johnson and Hulburt (1950), have shown that diffusion would be very important in preventing the formation of a stable layer at such a level. It is possible that the diffusion will be very much reduced by the collisions of the ions with other ions, which was neglected by both Ferraro, and Johnson and Hulburt, but the difficulty of forming the layer remains and is more important when it is remembered that F_2 region is sometimes found at height of 300 km and above.

The rocket results at high altitudes thus imply, if they are to be accepted, a very drastic revision of the admittedly unsatisfactory theory of the F_2 region; it will not be possible to discuss phenomena at these levels in terms of a static atmosphere.

There is no evidence from the rocket results of any seasonal or daily variation of pressure at these levels and the few experiments that have been made at places other than White Sands (32°N) have shown no significant latitude dependence. Some serious disagreement between the densities deduced from usual meteor data and the rocket data have been interpreted as due to a possible latitude effect but more recent meteor reductions have decreased the discrepancy and also an apparent seasonal change in the meteor data (Jacchia 1952).

There do not appear at the present time to be any reliable experimental data indicating seasonal or diurnal variations of temperature or density at these levels; the data are however insufficient to disprove the existence of effects large enough to have important ' meteorological ' consequences.

The comparatively low values of temperature and temperature gradient found by the rocket techniques are in accordance with the gradients to be expected if the only heating of F_2 were due to the ionization process which is observed (i.e., a rate of ion production of order αn_e^2, where α is the recombination coefficient) and the cooling due to molecular conduction to E region. La Gow estimates (following Bates) that the gradient would be 12°K/km at sunspot maximum and 0·5°K/km at sunspot minimum. It is however impossible to discount other heating processes. It does seem certain that the levels above 150 km are cooled by conduction rather than by radiation as had been assumed by many workers in theoretical calculations.

5. Winds and movements in the upper atmosphere

The preceding sections have been concerned with the properties of the upper atmosphere considered as essentially a static system. Although to the accuracy of the data there was no evidence for differences in density or pressure over the globe, it seems likely that small but important differences do occur. It is known for example that the whole of the ultra-violet emission from the sun, of wavelength less than 2,000 Å, is absorbed in the part of the atmosphere above 100 km. Although the energy in this part of the solar spectrum is not known with any accuracy, it is probably of the order of 1 erg sec^{-1} per cm^2 column and in the absence of cooling mechanisms this would lead to an appreciable heating of the atmosphere (of the order 1°K/hr). Thus it would require efficient cooling mechanisms to prevent daily and seasonal changes of temperature of the order of a few degrees.

Until more detailed systematic measurements of pressure and density are available it would be reasonable to suppose that temperature and hence pressure differences will exist and that they will give rise to winds in the high atmosphere in accordance with the

usual meteorological laws, with the viscosity playing an important part at the low densities involved.

In some particulars the meteorology of the upper air is very different from that of the troposphere, where it is very unusual for the wind to reverse over a wide area in a time short compared with a day. It is well known that there is at the ground a small but very regular oscillation of the barometric pressure with a period of 12 hr, the so-called solar atmospheric tide, and a still smaller lunar atmospheric tide. The pressure distribution of the solar tide must be associated with a wind system which in middle latitudes could be represented by a wind vector, of order 30 cm/sec, which rotates in direction with a period 12 hr; the wind blows towards the west at 10 LMT.

The theory of this type of pressure oscillation in the atmosphere has been developed by Pekeris (1937), Wilkes (1949) and others who have shown that its importance is likely to increase with height in the atmosphere. Numerical calculations suggest that at 100 km the amplification of the wind velocity would be about 200, i.e., the velocity would be about 50 m/sec; the exact value is uncertain since it depends upon the actual variation of temperature with height and also the theory is nearing the limit of its applicability near 100 km. It may be noted that an amplification exceeding 800 is quite impossible for this type of oscillation since it would imply a pressure oscillation greater than the static pressure.

Thus, in the upper levels of the atmosphere, winds having a gravitational origin such as the solar tidal wind are likely to be relatively far more important than at the ground.

Some evidence for these tidal wind circulations has long been given by the well known daily variation of the earth's magnetic field. The various components of the magnetic field show solar and lunar daily variations which are consistent with a dynamo theory based on the effects of a semi-diurnal system on the conducting layers of the ionosphere (Chapman and Bartels 1940). This theory is the only one so far advanced to account for the lunar variations and, provided the air velocities in the wind systems are about 200 times those inferred from the pressure oscillations at the ground, the observed conductivity of the ionosphere is about adequate to account for the observed effects. In the case of the solar magnetic variation it has been shown that the electric currents causing the effect almost certainly flow at a height of about 100 km, by direct rocket measurements of the field at different heights (Singer, Maple and Bowen 1951).

6. Measurements of winds in the high atmosphere

A number of methods have been used to determine more or less directly the wind velocity at different heights; most methods are cumbersome and expensive, have only been used on a few occasions and cannot give the synoptic data which are highly desirable in these problems.

Observations of the drift of smoke clouds from an exploding shell allowed determinations of the wind at 30 km to be made for a considerable time. The winds observed showed a seasonal variation, the predominant wind was from E to W during the summer months (in SE. England) and from W to E in the winter months (Murgatroyd and Clews 1949).

The study of the abnormal propagation of sound to great distances and the propagation of sound from a projectile exploded at a height allows an estimate of the wind to be made in addition to the scale height, provided the sound is received at a sufficient number of points. These various experiments have shown that between 30 and 60 km the wind velocity is also roughly from E to W during the summer and reversed during the winter.

The rocket results gave wind velocities which increased with height from 30-55 km where the velocity was of the order 100 m/sec. The velocity in summer appeared to be rather less than the velocity in winter (Ference 1952).

Above 55 km the evidence for wind velocities is inferred entirely from naturally occurring phenomena. At about 80 km noctilucent clouds are sometimes observed, during the period between sunset (or sunrise) at the ground and at that level. These clouds appear to drift from E to W and to show a structure suggestive of some turbulence. The observed wind velocities are of the order 50 m/sec; these clouds are seen rather rarely and so give little idea of the variations of wind velocity or direction and they are also only visible for relatively restricted times of the day.

Bright meteors occasionally leave a train which persists for some time, in spectacular cases for 30 min or more, but generally for 10-60 sec. Observations of the movements of these meteor trains against the background of the fixed stars allow the velocity of drift to be estimated. A characteristic feature of these trains is that although they start as straight trains they rapidly become distorted, implying a considerable wind shear in this part of the atmosphere. Generally, observations of persistent meteor trains have been spasmodic; a large amount of data was collected by Olivier but the data appear to be rather unsystematic; they serve to indicate that there are winds of the order 100 m/sec between 80-120 km and that there is considerable wind shear. More recently Whipple and his collaborators (Liller and Whipple, ref. G. 4) at Harvard have started systematic observations on meteor trains of relatively short duration, so that more trains are available. The meteors are observed by photography from three sites with super-Schmidt cameras. Analysis of the photographs allows the train to be located and followed accurately.

They find, from a detailed study of 5 such trains, wind velocities of the order 50-100 m/sec and a very large wind shear with altitude, the maximum shear observed being of the order 50 m sec^{-1} km^{-1}. According to these results the wind velocities are uncorrelated at points separated 5 km in height, but the wind appears to vary less rapidly in the horizontal plane. Thus the meteor observations suggest a rather turbulent atmosphere between 80 and 110 km. Since the theory of the persistence of a meteor train is obscure, it is not certain that this situation is characteristic of the undisturbed atmosphere or whether part at least of the observed movement is the result of the effect of the rather large meteorite on the air.

7. Movements of ionization in the upper atmosphere

Radio methods of investigation of the upper atmosphere are concerned mainly with the distribution of ionization and particularly with the distribution of free electrons. By using these methods it is possible to detect movements of ionization density in the atmosphere at various heights between 80 and 400 km.

The most direct application of this type of method is the study of the movement of the ionization left by a meteor ; the passage of a meteor through the atmosphere produces a train of ionization which remains for a time generally of the order of a few seconds before it is dissipated by diffusion and recombination. These meteor trains are detectable by radio methods which are very similar to radar techniques. The meteor train reflects strongly radio waves incident normal to the direction of the train; a study of the changes in range of the train gives its radial velocity and if sufficient meteors are observable at all directions from the radio set, the drift velocity of the trains can be found in magnitude and direction. Manning *et al.* (1950) have observed the drift velocity by measuring the Doppler beat between the radio wave reflected from the train and the emitted wave. Greenhow (1952) has obtained the same information by an accurate study of the range

of the train. This work on the movement of ionized meteor trains is very similar to the study of visual trains but has the great advantage that very much smaller meteors may be studied so that it is possible to obtain data at all times by observations of sporadic meteors.

Some evidence of turbulent velocities is provided by a study of the echo from the train. If the meteor train is straight, the echo is a simple reflection of the emitted pulse, but as soon as the trail becomes bent or broken up the reflected echo becomes complex. In general it is found that the echo does become complex and its amplitude begins to fluctuate within a few tenths of a second, from which it is deduced that there is appreciable velocity shear and that the velocities at points separated by a few kilometres are uncorrelated; this result is in agreement with the more detailed deductions from the visible meteor trains.

The results for the steady drift velocities obtained by this method agree well with those obtained by the other ionospheric methods and will be discussed with them.

The study of irregularities of ionization in the normal ionospheric regions is slightly less direct. In the presence of free charges the refractive index of the air becomes complex so that the phase velocity of the wave is increased and in general there is some attenuation; the group velocity in the medium is reduced. Thus if there is a horizontal layer of the ionosphere in which there are irregularities in electron density, the radio wave after passing through this layer will vary in phase and amplitude across the horizontal plane, in the same way as would happen after passage through a diffracting screen.

The wave reflected after passing through such an irregular ionosphere would thus give rise at the ground to a diffraction pattern and the echo amplitude would vary over the ground. If the ionospheric irregularities were drifting as the result of a ' wind ' the diffraction pattern on the ground would also move with little change in form; if however the irregularities were to move randomly, or to disappear and appear irregularly, the diffraction pattern on the ground would vary randomly and the changes at two places would be uncorrelated. When the fading of an echo from the ionosphere is observed, (using radio frequencies between 2 and 6 mc/s), it is found that the fading becomes uncorrelated for points separated by more than about a wavelength; at closer spacings the fading is very similar but often the characteristic features of the fading pattern do not occur simultaneously but separated in time by a small amount, which is the result expected if the ionospheric irregularities are drifting about. Measurement of these time delays for three suitably placed points enables the drift velocity to be estimated.

This method of studying drifts of ionization can be applied on almost all occasions; it only fails when the ionosphere is very smooth and hence the fading is very slight. It has the disadvantage that the irregularity causing the fading might be anywhere below the reflection point so that the height of the measured drift velocity is uncertain; it is likely however that the important region is near the reflection point where a small change in refractive index (which is nearly zero) has a marked effect.

This method of studying ionization movements has been applied to reflected waves from the E and F regions of the ionosphere and also to the highest part of the F region which contains irregularities which cause ' twinkling ' of point sources of radio waves outside the solar system (Hewish 1951).

Occasionally irregularities are observed at about the height of region E which give discrete echoes and in this case these irregularities may be followed and their velocities determined.

In the F region there are often quite large irregularities which show themselves as alterations in the distribution of electron density with height and alter the shape of the observed relation between equivalent height and radio frequency which is studied at ionospheric observatories. These irregularities also appear to move from place to place so that they may be used to determine a velocity of drift (Beynon 1948, Munro 1950).

The results obtained by the various methods appear to be in good agreement. Results from the fading method have now been obtained at the Cavendish Laboratory for a period of 4 years (Phillips 1952). Similar series of observations are also being taken at Washington (Salzberg and Greenstone 1951) in New Zealand and in Canada (Chapman 1953). These various results show a consistent picture of the ionospheric drift particularly in region E. The drift velocity shows a considerable daily variation and marked seasonal changes.

The average drift speed is about 80 m/sec and for most of the time lies between 20 m/sec and 150 m/sec. There is a marked semi-diurnal component in the E to W and N to S components of velocity; the semi-diurnal drift vector appears to be about 40 m/sec and to rotate in direction in the way expected for the tidal oscillation wind vector. The drift is from E to W at midday. In the winter and equinoctial months the semi-diurnal drift vector seems to be predominant but in the summer months there is a diurnal component corresponding to a drift from W to E at noon and this drift velocity is larger than the semi-diurnal component so that the resultant drift at midday is from W to E in summer and from E to W in winter and at the equinoxes. There is some evidence that this result also applies in the southern hemisphere so that the effect is truly seasonal.

Chapman, working in Canada, has analysed the drift velocities for a lunar effect and has found one of amplitude about 20 m/sec corresponding to a drift from E to W at lunar transit; this effect has not so far been confirmed from observations elsewhere. The ratio of lunar to solar drift tides is larger than that observed at the ground or in the magnetic variations.

In F region the fading method indicates that the velocities are of the same order of magnitude as in E region; the scintillations of radio stars, which occur only during the night, indicate rather higher velocities at the top of the region; there is no steady rotation of the velocity vector of the type found in E region. The drift is mainly to the west in the night time and to the east in the day with a rather rapid changeover. The N-S component of the drift is more variable in behaviour although its average magnitude is comparable to the E-W.

A marked dependence of the drift velocity in the F region, as deduced by scintillations and by fading methods, upon the magnetic disturbance K figure has been found. At the highest levels the drift velocity depends nearly linearly on the K figure and at times of magnetic storm may reach 800 m/sec. There is some evidence that this drift velocity changes from being westward before midnight to eastward after midnight. In the lower part of F region the drift is much enhanced for K figure exceeding about 3 but appears to be mainly towards the west, there is no evidence for a reversal of direction at midnight.

In E region there appears to be an enhancement of drift velocities for high K figures above about 6; there are as yet too few data to settle whether these increases depend upon time of day in any regular manner.

A detailed survey of all the observational data has been made by Briggs.

With the exception of the radar tracking of meteor trains, all the methods of studying these ionospheric drifts are concerned with the movements of regions of electron density differing, probably only slightly, from the surrounding ionosphere. These movements may arise from a number of causes.

(1) It is possible that an extra ionizing agent is acting all the time and is moving through the atmosphere. This seems a very unlikely event but cannot be dismissed entirely.

(2) The variations of ionization density may be due to a type of magneto-hydrodynamic wave in the ionized air. The measured velocity would then be the group velocity of the waves. Waves of this type have been studied in some detail by Hines (1953).

(3) The drift may be a drift of the irregularity of ionization through the atmosphere and may be related in a very complicated way to the movement of the actual neutral air.

(4) The measured velocities may be indications of ordinary winds in the air, the neutral air and the ionization with its irregularities moving together.

Movement of ionized gas in the upper atmosphere is controlled largely by the magnetic field of the earth. If a wind tends to blow the ionized gas across the magnetic field, then the charges must experience an induced electric field perpendicular to the wind velocity and the magnetic field. In a very rare extended gas, the ionized air would be quite unable to move with the air across the magnetic field and a current would flow. In the presence of neutral particles, collisions between the ions and the neutral particles allow some movement with the air, to an extent depending upon the ratio of the frequency of collisions to the frequency of gyration about the magnetic field.

In a gas of finite extent, the flow of current may be prevented and a polarization electrostatic field may be set up which, in the extreme case, could completely annul the effect of the magnetic field.

In any particular case the problem becomes a very complicated discussion of what currents may flow and what polarisation fields may be set up. A typical example of the complexity of the problem is given by the dynamo theory of the daily variations of the magnetic field (Baker and Martyn 1952, Hirono 1952). These considerations apply to the main bulk of the ionized gas; the effect on an irregularity of ionization is still more complicated. A region of high conductivity will take an extra current density and will hence have a different interaction with the magnetic field from the neighbouring normal ionized gas and it may be expected to move through the ionized gas. No satisfactory theory of the interpretation of the movement of irregularities of ionization has as yet been offered.

If the main ionization of the upper regions is in motion as a result of induced electric fields, the collisions between the drifting ions and the neutral air particles will tend to set up a movement of the neutral air, so that it is possible that air winds may be set up by currents in the ionized air. If the density of positive charges is n_+ cm^{-3} and their mass and drift velocity are m and u, then the drift momentum destroyed at each collision is mu and the momentum given to the air per sec is $n_+ muv$, where v is the collision frequency. Hence the air will experience an acceleration $n_+ muv/\rho$ or approximately vun_+/n. Thus, if an appreciable part of the air is ionized, the effect on the neutral air may be very considerable. For F region n_+/n may be as large as 10^{-3} and v 10^{-1} so that the air would itself reach velocity u in a time of the order 10^4 sec. Such an induced wind in the air would complicate the process by which the ionic drift was set up, but the calculation serves to show that in the ionized regions the air may be set in motion by drifts of the ionization. Considerable caution is thus necessary in interpreting the measured drift velocities of ionization irregularities in terms of air movements.

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Computation of tendencies and vertical motion with a two-parameter model of the atmosphere

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SUMMARY

The system of partial differential equations given by Sawyer and Bushby (1953) for the rates of change of the 1,000-500-mb thickness and the 500-mb contour height have been solved on an electronic computing machine for three synoptic situations and the fields of vertical motion were also computed. The results, described in this paper, show reasonable agreement with actuality. The implied 1,000-mb height tendencies also agree quite well with those actually observed.

1. INTRODUCTION

In recent years, several attempts have been made to forecast by numerical methods the large-scale atmospheric motions. Charney (1949) justified the use of the geostrophic approximation in evaluating the horizontal velocity and the vertical component of vorticity after the horizontal divergence has been eliminated from the basic equations. Charney also showed that if one assumes that the wind is uni-directional with respect to height and that the increase of wind with height is the same along all verticals, then there is one level in the atmosphere (approximately in mid-troposphere) at which the flow corresponds to the flow in a barotropic fluid. Several writers, including Charney, Fjörtoft and von Neumann (1950) and Bolin and Charney (1951), have tested this theory by applying the vorticity equation relating to the motion of a barotropic fluid to the 500-mb isobaric surface in order to predict changes in the height of that surface. The degree of success achieved with this model of the atmosphere was not sufficient to justify using it to produce routine forecasts, but was enough to warrant extending the model to take account of some of the baroclinic properties of the atmosphere.

Phillips (1951) indicated a method of representing some of the simple baroclinic properties of the atmosphere by considering two barotropic fluids superimposed one upon the other, the whole being contained between rigid horizontal plates. Charney and Phillips (1953) gave a generalization of this in which the atmosphere is divided into n layers, the motion being assumed barotropic in each layer.

An alternative way in which some of the baroclinic features of the atmosphere can be incorporated in a simple model is to assume that the variation of the horizontal and vertical components of velocity with height is the same along all verticals. Eady (1952) gave the title of ' $2\frac{1}{2}$-dimensional model ' to this type of model atmosphere, and Eliassen (1952), Thompson (1953) and Sawyer and Bushby (1953) have derived sets of equations which describe the motion in particular versions of this model.

Sawyer and Bushby derived, subject to various approximations, a set of simultaneous partial differential equations for forecasting the contour height, h_m, of an isobaric surface representative of mid-troposphere and the thickness, h', of the layer from 1,000 mb to that isobaric surface. The method is one in which the instantaneous rates of change of h_m and h' are first computed at time t, and from these, new values of h_m and h' are computed for time $t + \delta t$. The process is then repeated n times, and forecast charts are produced for time $t + n\delta t$.

The construction of a programme for the solution of this set of equations by an electronic computing machine is a formidable task. It was therefore decided to prepare one programme for the solution of the equation for the rate of change of contour height and another for the solution of the equation for the rate of change of thickness before proceeding with the step-by-step process of time integration. The present paper describes the computation of the rates of change of h_m and h' for three synoptic situations, and the computed results are compared with the actual changes that occurred. The effect of assuming wrong boundary conditions is discussed, as is the effect of neglecting the vertical advection of vorticity.

2. The Sawyer-Bushby 2½-dimensional model

The model of the atmosphere adopted by Sawyer and Bushby (1953) consists of a baroclinic fluid in which the thermal wind is constant in direction in any vertical column, but not necessarily parallel to the wind direction at any level, and the thermal wind speed is assumed proportional to the pressure difference through the layer concerned. The atmosphere is considered bounded by two pressure surfaces across which the movement of air is assumed to be negligible. These surfaces are $p = p_1$ corresponding to the upper limit of the troposphere and $p = p_0$ corresponding to a level near the surface. Furthermore, the variation of vertical motion with respect to pressure is assumed to be parabolic. The geostrophic approximation is made after the horizontal divergence has been eliminated from the equations.

Conformally transforming the northern hemisphere by a stereographic projection from the south pole on to a plane perpendicular to the earth's axis and passing through the north pole, the following equations were derived for determining $\partial h_m/\partial t$, $\partial h'/\partial t$ and Π where $\Pi = dp/dt$ is a measure of the vertical motion :

$$\nabla^2 \frac{\partial h'}{\partial t} + J(h_m, \beta^2 g l^{-1} \nabla^2 h') + J(h', \beta^2 g l^{-1} \nabla^2 h_m + l) = \frac{4\Pi_m (\nabla^2 h_m + l^2/\beta^2 g)}{(p_0 - p_1)} \quad . \quad (1)$$

$$\nabla^2 \frac{\partial h_m}{\partial t} + J(h_m, \beta^2 g l^{-1} \nabla^2 h_m + l) + \tfrac{1}{3} J(h', \beta^2 g l^{-1} \nabla^2 h') - \frac{8\Pi_m \nabla^2 h'}{3(p_0 - p_1)} = 0 \quad . \quad . \quad (2)$$

$$\frac{\partial h'}{\partial t} = \beta^2 g l^{-1} J(h', h_m) - RA \Gamma_p \Pi_m / g \quad . \quad . \quad . \quad (3)$$

where

$$A \equiv \tfrac{1}{2} \frac{p_0 + 3p_1}{p_0 - p_1} - \frac{4p_0 p_1}{(p_0 - p_1)^2} \log_e \frac{2p_0}{p_0 + p_1} ;$$

∇^2 and J are the two-dimensional Laplacian and Jacobian operators referred to rectangular Cartesian coordinates on the plane projection of the earth's surface; β is the magnification factor $\sec^2(\pi/4 - \theta/2)$, θ being the geographical latitude; g is the acceleration due to gravity; l is the Coriolis parameter; Π_m is the value of Π at the pressure level p_m, the mean level of the layer considered; R is the universal gas constant and Γ_p a measure of the departure of the lapse rate from the adiabatic given by $\Gamma_p = \partial T/\partial p - \gamma/g\rho$, γ being the appropriate wet- or dry-adiabatic lapse rate, ρ the density and T the absolute temperature.

In the present work the values of p_0 and p_1 were taken to be 1,000 mb, and 200 mb respectively. This would give $p_m = 600$ mb, a level for which charts are not normally available. h_m was therefore identified with the height of the 500 mb surface and h' with the thickness of the 1,000-500 mb layer.

3. Method of solution

Eliminating Π_m from Eqs. (1) and (3) gives

$$\nabla^2 \frac{\partial h'}{\partial t} + \frac{4g\,(\nabla^2 h_m + l^2/\beta^2 g)\{\beta^2 g\, l^{-1} J(h_m, h') + \partial h'/\partial t\}}{RA\, \Gamma_p\, (p_0 - p_1)} + J(h_m, \beta^2 g l^{-1} \nabla^2 h') + J(h', \beta^2 g l^{-1} \nabla^2 h_m + l) = 0 \quad . \quad (4)$$

a Helmholtz type of differential equation in $\partial h'/\partial t$. Eqs. (4), (3) and (2) can be solved numerically to give values of $\partial h'/\partial t$, Π_m and $\partial h_m/\partial t$ at a finite number of points, given the values of $\partial h'/\partial t$ and $\partial h_m/\partial t$ along the boundary of the area for which the solution is carried out. The following finite-difference approximations were used in evaluating the Jacobian and Laplacian operators :

$$(\nabla^2 z)_0 = \frac{1}{a^2}\left[\sum_1^4 z_1 - 4z_0\right] \quad . \quad . \quad . \quad (5)$$

and

$$[J(x, y)]_0 = \frac{1}{4a^2}(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3) \quad . \quad . \quad (6)$$

where the suffixes refer to the grid points shown in Fig. 1 and where a is the grid length. The grid length used in the present computations was about 162 statute miles in middle latitudes and a grid of 16 × 12 points was used. This enables values of $\partial h'/\partial t$ and $\partial h_m/\partial t$ to be computed at 12 × 8 points (excluding boundary points) as two rings of points are 'lost' in calculating the third-order differences necessary to evaluate the vorticity advection terms.

Eqs. (2) and (4) can be solved by the Liebmann iterative process. As Eq. (4) is a Helmholtz type of equation its evaluation converges more rapidly than the solution of Eq. (2), a Poisson type of equation. Twenty iterations were necessary to obtain the solution of Eq. (4) to the required degree of accuracy whereas 40 iterations were necessary

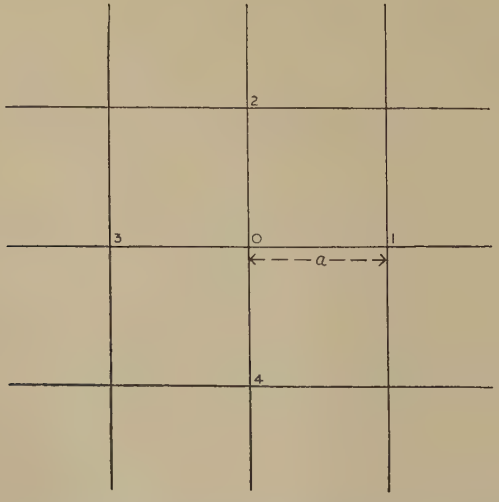


Figure 1. The grid used in evaluating the Jacobian and Laplacian operators by finite-difference approximations.

to obtain the solution of Eq. (2). The 1,000-mb height tendency, $\partial h_0/\partial t$, was calculated by hand from the following relationship :

$$\frac{\partial h_0}{\partial t} = \frac{\partial h_m}{\partial t} - \frac{\partial h'}{\partial t} \quad . \quad . \quad . \quad . \quad (7)$$

LEO, an electronic computing machine owned by Messrs. J. Lyons & Co. Ltd., was used to solve Eqs. (4) and (3) for $\partial h'/\partial t$ and Π_m. The time taken was about 15 min of which 6 min was spent in reading programme and data, 4 min in printing results and 5 min in computing. A slightly shorter time was taken by the programme which solved Eq. (2) for $\partial h_m/\partial t$.

The field of Γ_p was evaluated by using a method similar to that used by Bushby (1952). The computations were then repeated using a constant value of Γ_p (38°F per 500 mb), but there was no significant change in the results. The boundary values of $\partial h'/\partial t$ and $\partial h_m/\partial t$ were assumed zero but after the computations had been made an estimate was made of the effect on the answer of using the actual boundary conditions that were observed. The solution of Eq. (2) was repeated using zero values for the vertical advection of vorticity term as it was thought that this term would not have an appreciable effect on the solution. Neglecting this term Eq. (2) becomes

$$\nabla^2 \frac{\partial h_m}{\partial t} + J(h_m, \beta^2 gl^{-1} \nabla^2 h_m + l) + \tfrac{1}{3} J(h', \beta^2 gl^{-1} \nabla^2 h') = 0 \quad . \quad (8)$$

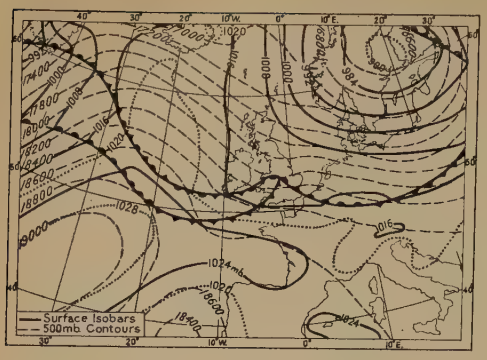
4. Results

The three synoptic situations for which computations were made are shown in Figs. 2a, 3a and 4a, and Figs. 2b, 3b and 4b are charts of the computed field of vertical motion, Π_m. The actual 500-mb height-tendency fields, obtained from the change of 500 mb height in a 12-hr period centred at the times for which the computations were made, are shown in Figs. 2c, 3c and 4c, whilst Figs. 2d, 3d and 4d show the values of the 500-mb height tendency computed by solving Eq. (2) assuming all the boundary values are zero. Similarly, the actual 1,000-500 mb thickness tendency fields, meaned over 12 hr, are shown in Figs. 2e, 3e and 4e, and the thickness tendencies computed from Eq. (4) in Figs. 2f, 3f and 4f. The actual 1,000-mb height-tendency fields, averaged over a 6-hr period centred at the times for which the computations were made, were measured from the surface charts and are shown in Figs. 2g, 3g and 4g. The computed fields of $\partial h_0/\partial t$, assuming zero boundary values, are shown in Figs. 2h, 3h and 4h.

It can be seen from the diagrams that there is a good correspondence between the computed and actual tendency fields. At first, it may seem inconsistent that an equation for $\partial h_m/\partial t$ in which the main terms are those of Charney's equation for a barotropic model atmosphere gives much better results than those found by Charney (1949) or Bushby (1951) when solving the barotropic equation. The additional terms are obviously not responsible for all the improvement, and it therefore seems that the use of a much smaller grid length is an important factor. The 450-mi grid length used in previous work by Charney and Bushby appears to be much too coarse to represent adequately the vorticity field by finite-difference approximations, and Charney *et al.* (1950) came to a similar conclusion.

Correlation coefficients were calculated comparing the computed tendencies with the actual tendencies, and regression coefficients were calculated for the best-fitting straight line giving the actual rate of change (Y feet per hour) in terms of the computed rate of change (X feet per hour) in the form

$$Y = bX + c \quad . \quad . \quad . \quad . \quad (9)$$



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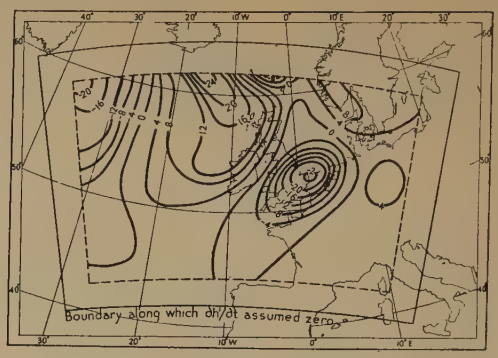

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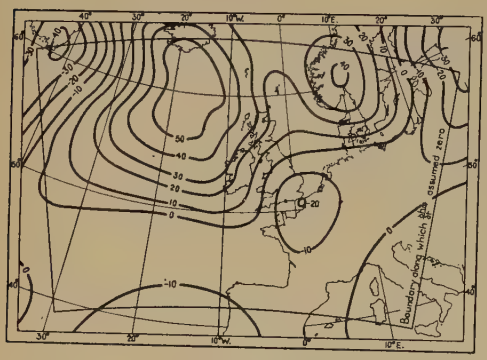

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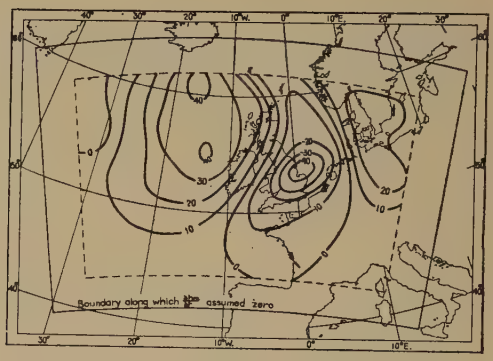

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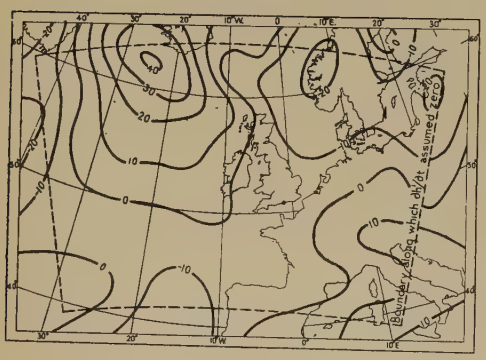

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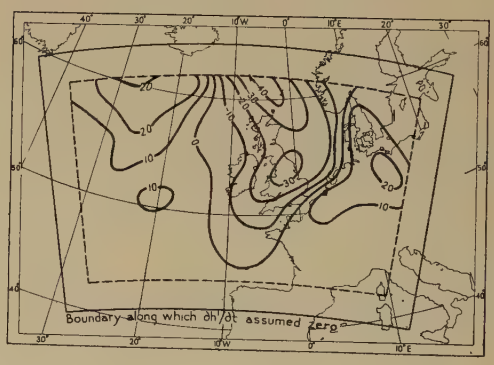

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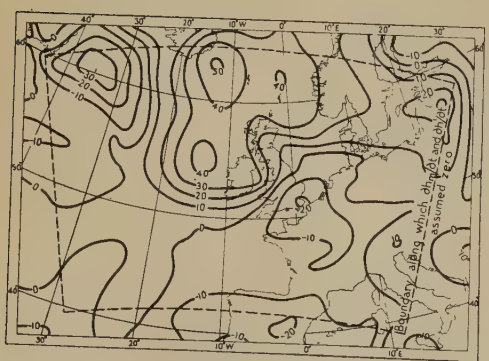

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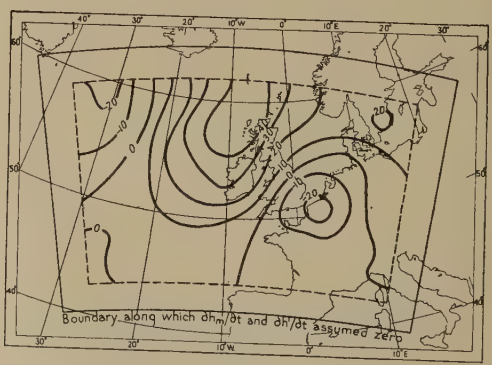
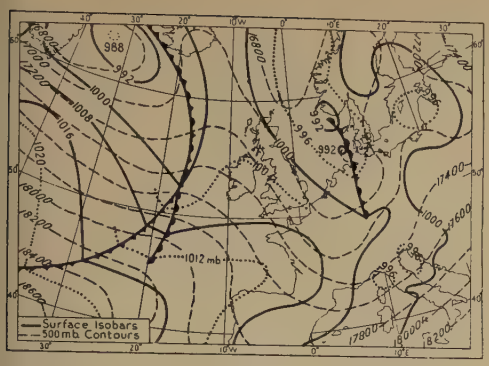
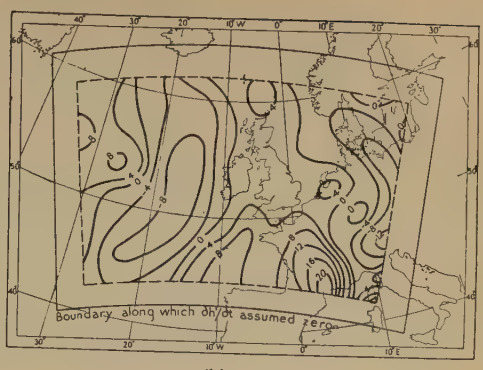


(h)

Figure 2. 15 GMT 14 March 1949. (a) Synoptic chart; (b) Vertical motion (Π_m in mb/hr); (c) Actual 500 mb height tendency (ft/hr); (d) Computed 500 mb height tendency (ft/hr); (e) Actual 1,000-500 mb thickness tendency (ft/hr); (f) Computed 1,000-500 mb thickness tendency (ft/hr); (g) Actual 1,000 mb height tendency (ft/hr); (h) Computed 1,000 mb height tendency (ft/hr).



(a)



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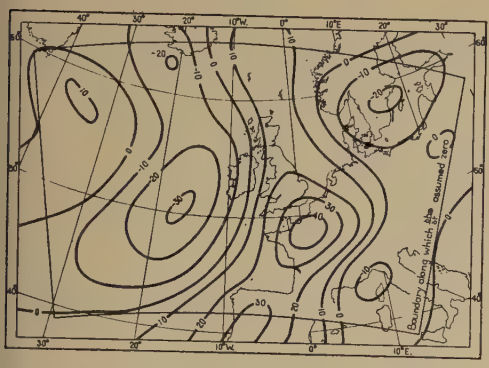

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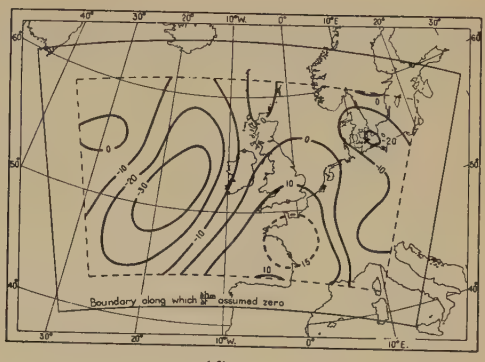

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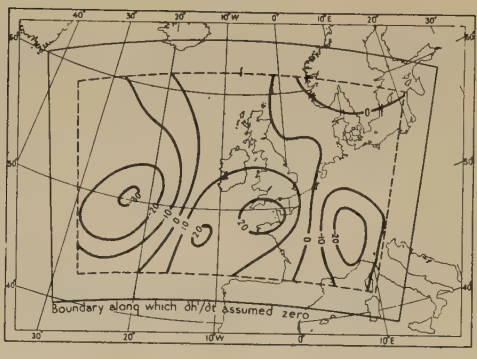

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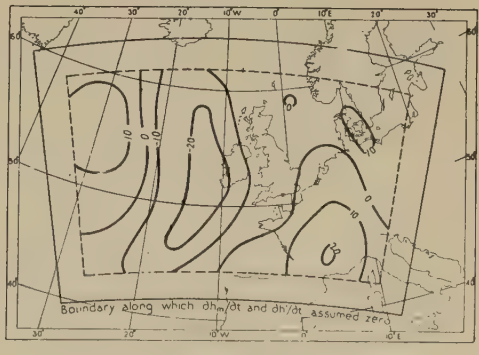

(h)

Figure 3. 15 GMT 27 January 1952. (a) Synoptic chart; (b) Vertical motion (Π_m in mb/hr); (c) Actual 500 mb height tendency (ft/hr); (d) Computed 500 mb height tendency (ft/hr); (e) Actual 1,000-500 mb thickness tendency (ft/hr); (f) Computed 1,000-500 mb thickness tendency (ft/hr); (g) Actual 1,000 mb height tendency (ft/hr); (h) Computed 1,000 mb height tendency (ft/hr).

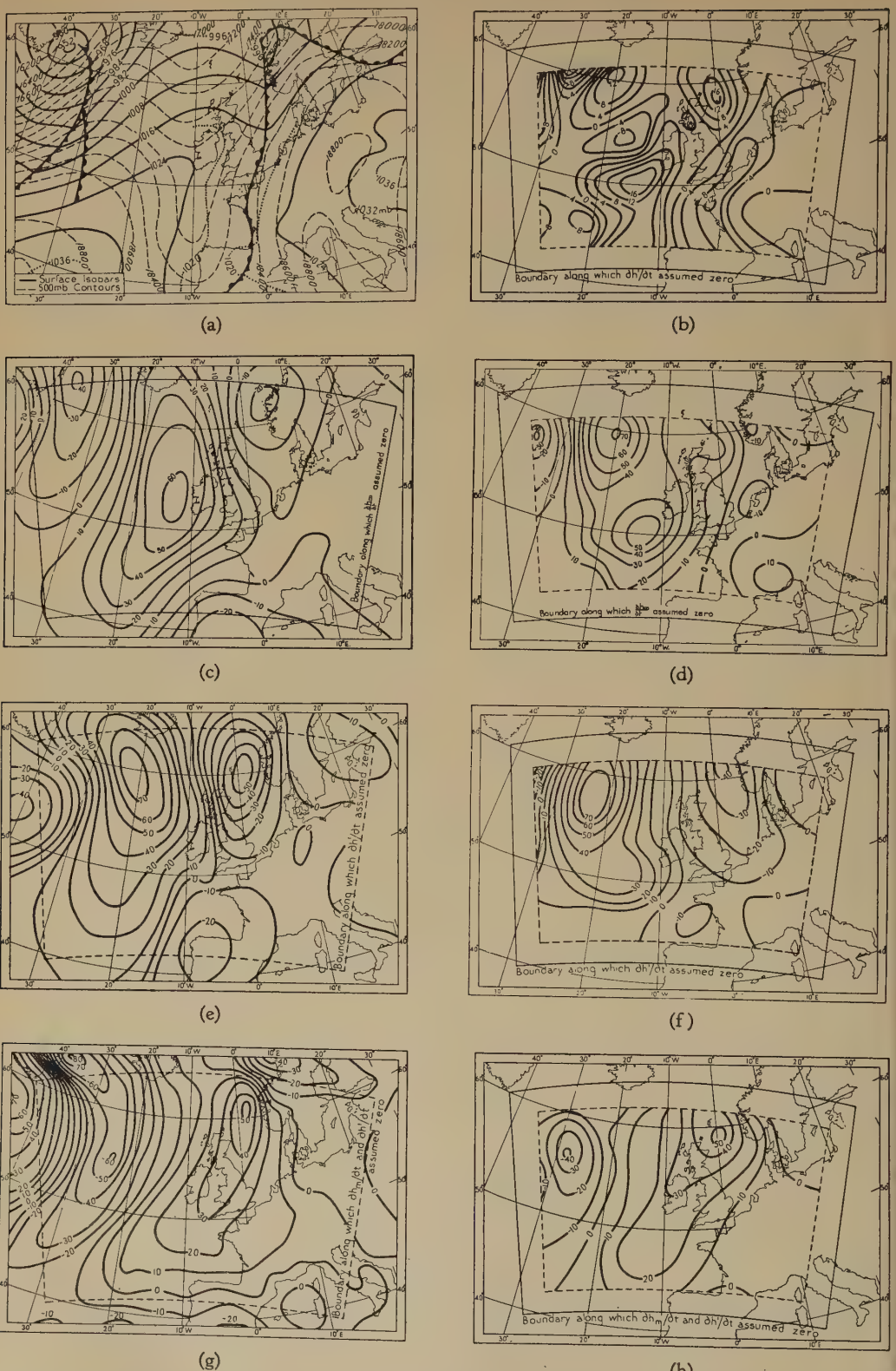

Figure 4. 15 GMT 20 December 1951. (a) Synoptic chart; (b) Vertical motion (Π_m in mb/hr); (c) Actual 500 mb height tendency (ft/hr); (d) Computed 500 mb height tendency (ft/hr); (e) Actual 1,000-500 mb thickness tendency (ft/hr); (f) Computed 1,000-500 mb thickness tendency (ft/hr); (g) Actual 1,000 mb height tendency (ft/hr); (h) Computed 1,000 mb height tendency (ft/hr).

and the results are shown in Table 1. The root mean square differences between the actual and computed tendencies are given in Table 2. The notation used in Tables 1 and 2 is as follows :

A = Actual $\partial h_m/\partial t$ (ft/hr) obtained from change in h_m over 12-hr interval.

B = Computed $\partial h_m/\partial t$ (ft/hr) using Eq. (2), zero boundary values.

C = Computed $\partial h_m/\partial t$ (ft/hr) using Eq. (8), zero boundary values.

D = Computed $\partial h_m/\partial t$ (ft/hr) using Eq. (2), actual boundary values.

E = Computed $\partial h_m/\partial t$ (ft/hr) using Eq. (8), actual boundary values.

F = Actual $\partial h'/\partial t$ (ft/hr) obtained from change in h' over 12-hr interval.

G = Computed $\partial h'/\partial t$ (ft/hr), zero boundary values.

H = Computed $\partial h'/\partial t$ (ft/hr), actual boundary values.

J = Actual $\partial h_0/\partial t$ (ft/hr), obtained from change in surface pressure over 6-hr interval.

K = Computed $\partial h_0/\partial t$ (ft/hr), zero boundary values.

L = Computed $\partial h_0/\partial t$ (ft/hr), actual boundary values.

TABLE 1. CORRELATION AND REGRESSION COEFFICIENTS OF 500-MB HEIGHT TENDENCIES, 1,000-500 MB THICKNESS TENDENCIES AND 1,000-MB HEIGHT TENDENCIES.

		15 GMT 14.3.49			15 GMT 27.1.52			15 GMT 20.12.51		
Y	X	r	b	c	r	b	c	r	b	c
A	B	0·64	0·72	6·2	0·78	0·99	9·8	0·69	0·78	7·2
A	C	0·67	0·85	3·1	0·83	1·05	5·6	0·70	0·76	4·3
A	D	0·82	0·74	4·1	0·80	0·85	6·2	0·77	0·76	7·1
A	E	0·84	0·80	1·4	0·83	0·85	2·2	0·77	0·74	4·3
F	G	0·49	0·39	− 0·4	0·76	0·67	1·5	0·85	0·94	2·4
F	H	0·72	0·49	0·2	0·80	0·61	− 0·1	0·90	0·80	2·2
J	K	0·93	0·98	− 0·3	0·67	0·72	− 0·9	0·83	1·18	− 8·0
J	L	0·95	0·68	− 0·1	0·78	0·73	2·7	0·88	1·07	− 1·3

r = correlation coefficient

TABLE 2. ROOT MEAN SQUARES IN FT/HR.

	15 GMT 14.3.49	15 GMT 27.1.52	15 GMT 20.12.51
A	21·3	15·8	26·5
$A-B$	16·3	14·0	17·5
$A-C$	14·5	10·3	16·6
$A-D$	12·5	12·2	16·1
$A-E$	11·1	9·5	15·4
F	10·5	10·6	27·3
$F-G$	12·3	8·9	13·7
$F-H$	10·8	8·3	12·7
J	17·1	12·0	22·7
$J-K$	5·9	9·2	14·6
$J-H$	9·2	9·4	10·7

It is evident from Table 1 that there is a significant correlation between the computed and actual tendency fields, this being greater when the correct boundary values were used. The results are confirmed by Table 2 for two of the situations, but for 15 GMT 14 March 1949 the root mean square of (F-G) is greater than that of F and the root mean

square of $(J\text{-}K)$ is less than that of $(J\text{-}H)$, which may at first seem inconsistent with the correlation coefficients of Table 1. However, this particular situation was one in which a rapidly-moving wave depression, followed by a mobile ridge, was moving ESE across the British Isles and eastern Atlantic and one would therefore not expect the computed instantaneous rates of change to be of the same magnitude as the measured rates of change meaned over a six- or twelve-hour period, although the tendency patterns should correspond. This point emphasizes the fact that it is difficult to obtain a satisfactory objective method of comparing two sets of isopleths of this type.

The values of the three vorticity-advection terms of Eq. (2) were printed by the machine during the course of the computations so that their relative importance could be assessed. In each case the term $J(h_m, \beta^2 gl^{-1} \nabla^2 h_m + l)$, which represents the horizontal advection of the 500 mb absolute vorticity field, is the most important. The term $\frac{1}{3} J(h', \beta^2 gl^{-1} \nabla^2 h')$ adds a contribution at about 60 per cent of the grid points whilst the last term, representing vertical advection of vorticity, makes a contribution at approximately 30 per cent of the grid points. Either of these last two terms can, in certain limited areas, be numerically greater than the first Jacobian. Ignoring the term representing the vertical advection of vorticity when computing $\partial h_m/\partial t$ did not significantly affect the correlation coefficients but decreased the root mean square difference between the actual and computed tendency, although in two of the cases the decrease was only slight. Here again, however, the root mean square differences may give a false impression owing to the use of 12-hr mean tendencies for comparison, and the effect of ignoring the vertical advection of vorticity can only be properly assessed after a time integration has been carried out.

It seems quite obvious from the available information that the assumption of the correct boundary conditions does significantly increase the agreement between the computed and actual tendency fields and it will be necessary to take some notice of this when attempting time integration.

The field of vertical motion computed for 15 GMT 14 March 1949 indicates an area of ascending motion ahead of the mobile wave depression, with a large region of descending motion ahead of the ridge over the eastern Atlantic. Another area of ascending motions is shown ahead of the central Atlantic troughs. In this situation the computed vertical motion agrees quite well with that expected from synoptic experience. The main feature of the computed vertical motion field for 15 GMT 27 January 1952 is the extensive region of subsidence indicated over southern France and the Bay of Biscay. This was associated with a surface-pressure ridge and is on the western side of a cold trough. Other features of interest are the areas of convergence ahead of the frontal troughs in the eastern Atlantic and southern Scandinavia. The vertical-motion field computed for 15 GMT 20 December 1951 is extremely interesting as the twin centres of subsidence to the east of Scotland and to the south-west of Ireland, with an area of convergence over western France, are symptomatic of the cutting-off process and anticyclonic building across the neck of a cold trough. This development actually occurred.

5. Conclusion

The main conclusion of the present work is that the Sawyer-Bushby model does give a good representation of the 1,000-500 mb thickness changes, the 500-mb contour-height changes and the field of vertical motion in the three situations to which it has so far been applied. Another, and unexpected result, is that the computed 1,000-mb height tendencies, obtained by subtracting the thickness tendencies from the 500-mb height tendencies, should show such good agreement with actuality.

The effect of boundary conditions seems important if a small area is considered, and before time integration is undertaken it would seem necessary either to increase the area under consideration so that the effect of boundary conditions would not affect the central area, or to make a preliminary estimate of expected changes around the boundary and feed this into the machine.

The present results appear to justify an attempt to produce prognostic charts by the simultaneous time integration of the contour and thickness-tendency equations.

Acknowledgments

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Turbulence in clouds as a factor in precipitation

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Summary

Visual and radar observations of the height of formation of precipitation in shower clouds in relation to temperature, and measurements of droplet population, show that when cumulus clouds produce showers it is often by coalescence, rather than by an ice-phase process.

Langmuir's work gave the conditions necessary for collisions to take place between droplets. The droplets of young cumulus clouds are less than 20 microns in radius and calculation shows that they are too small and settle too slowly by gravity alone to grow at an appreciable rate by colliding with each other.

The updraught in a cumulus cloud is highly turbulent so that the air containing the droplets is being perpetually accelerated in a random way. An analysis of random acceleration in one dimension shows that it would produce collisions if violent enough, and in conjunction with gravity would start droplets into growth. Once their radii reached about 20 microns they would continue to grow with or without turbulence. Such a mechanism is consistent with the observed changes in droplet size distribution.

Provided that sufficient energy were available in a suitable frequency range, turbulence would initiate a shower in a cumulus cloud. Numerical confirmation of the mechanism awaits suitable measurements of small-scale turbulence in cloud, but meanwhile an alternative approach through macroscopic energy considerations might prove fruitful.

1. Introduction

The onset of precipitation in a cumulus cloud originates at a point considerably below the top, at a temperature of about + 5°C, according to Battan (1953), and spreads rapidly in vertical extent. It is rather difficult to explain the process as observed, and particularly its rapid initial development, by a condensation and gravitational action alone. There would seem to be a possible role for turbulence to play in the process. Certainly strong turbulence is present in these clouds.

Glaciation can play a significant part, but it is not always present, and the range of heights at which glaciation can be expected does not seem to coincide with the region where precipitation is first observed by radar. Indeed, notably in tropical regions, rain often falls from clouds which lie entirely below the 0°C level, as reported by Hunt (1949), Virgo (1950), and E. J. Smith (1951). If droplets of radius 50 microns or of that order, or ice particles of comparable size, are assumed to exist in clouds, their calculated rapidity of growth is satisfactory, or at any rate of the right order of magnitude. To explain the rapid spread in vertical extent that is observed, it is probably necessary to assume the appearance of drops of such size almost simultaneously over a considerable range of heights.

While other quite reasonable approaches to the problem of growing these initial large droplets have been made (Ludlam 1951), consideration of turbulence as a possible cause of them seems justifiable. Gravity causes droplets of different sizes to approach each other, but seldom with sufficient speed to lead to coalescence; encounters under turbulence might be significantly more vigorous. Turbulence would make itself felt to a similar degree over a wide range of heights, and could perhaps be enhanced simultaneously throughout this range by the incidence of glaciation at or near the cloud top.

The possibility of turbulence playing some part in the precipitation process has been considered by other writers. Bergeron (1933) concluded that it would not be intense

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enough to do so in stratiform clouds, but did not discuss its action in cumuliform clouds. Arenberg (1939) considered the action of turbulence in causing water droplets to be thrown together, without taking account of the conditions necessary for them actually to collide. Mason (1952) introduced turbulence to prolong the paths travelled by droplets through a cloud in becoming drizzle, and Best (1952) to transport them to and from the centre of a cloud. Gabilly (1949) considered its effect in causing collisions, but confined his discussions to sinusoidal motion of the air, rather than true turbulence, which is a random motion.

In the present report, Gabilly's approach has been carried further to include the random character of turbulent motion, and to show its close relationship to gravity. A computation of the effect of a strong gravitational field on a cloud shows the possibly considerable effect which turbulence might have. Calculation of the effect of turbulence itself on such a cloud requires a knowledge, missing so far, of the strength and nature of turbulence inside a cloud. The work has been brought up to the stage where such a computation can be carried out as soon as the experimental data become available.

2. Clouds

In discussing the conversion of cloud into rain, it is necessary to know the initial condition of the cloud, and in particular, the size and number of the water droplets in the cloud. Direct measurements of these quantities have been made from aircraft, and described, by Diem (1942, 1948) and by Weickmann and aufm Kampe (1953). The droplets were caught in a film of oil and examined under a microscope. This method is incapable of detecting droplets of less than about 2 microns radius, but there may not be any to be detected. The method appears to be reliable for all droplets larger than

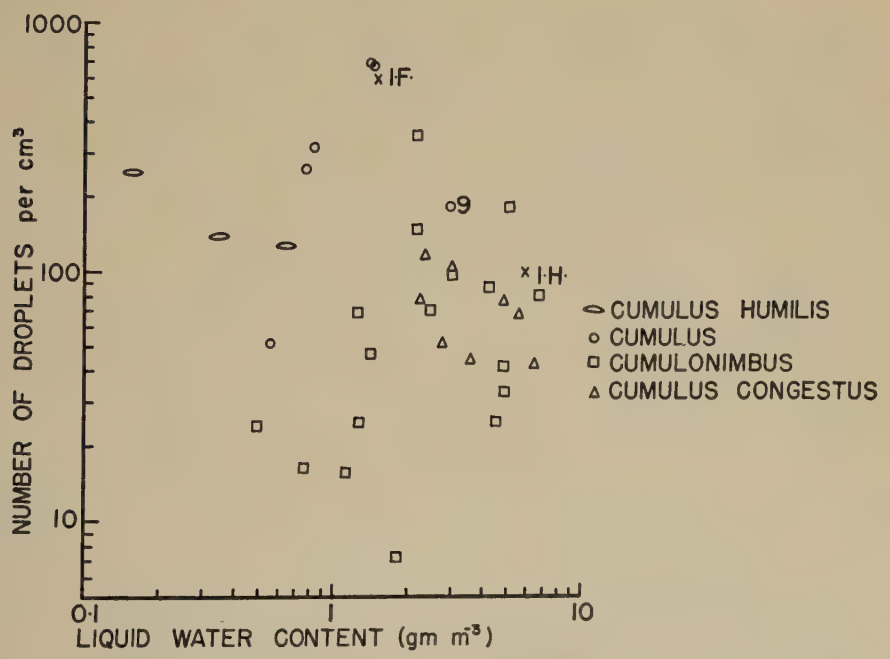

Figure 1. Properties of cumulus clouds (measured by Weickmann and aufm Kampe). Total number of droplets per cm³ is plotted against water content for each specimen. I.F. and I.H. represent idealized fair-weather and heavy cumulus characteristics respectively; for 9 see text.

this. Aufm Kampe and Weickmann measured only cumulus humilis, cumulus, cumulus congestus, and cumulonimbus; the first two classes can be conveniently grouped as fair-weather cumulus, and the last two as heavy cumulus. The data* include the distribution of the number of droplets with radius, together with the total number of droplets per cm^3 and the liquid water content, both deduced from visibility measurements. In Fig. 1, the droplet number is plotted against water content for the thirty-four samples. Fair-weather cumulus form a group with large droplet numbers and small water contents, clearly separated from heavy cumulus, which have one-fifth to one-sixth the number of droplets but about four times the water content. No. 9, however, appears to be an exception.

A survey of aufm Kampe's and Weickmann and Diem's distribution curves shows that, in most types of cloud, the number of droplets has a J-shaped distribution with radius, that is, there are a large number of small droplets and a small number of large droplets. In heavy cumulus, in particular, though most of the droplets are less than 20 microns in radius, a small but significant number of droplets have radii up to and beyond 40 microns. Thus these clouds must be regarded as ones in which some precipitation process has already started, so that rain or drizzle will soon fall (though it may evaporate before reaching the ground). However, fair-weather cumulus clouds are free from these larger droplets. In this paper they will be taken as the starting point for the calculations on coalescence processes.

The drop size distributions of a number of cumulus clouds have been replotted on logarithmico-normal charts (as described by Kottler (1950, 1951, 1952)). Some of the plots for fair-weather cumulus are reproduced in Fig. 2 (a). The steepness of the curves

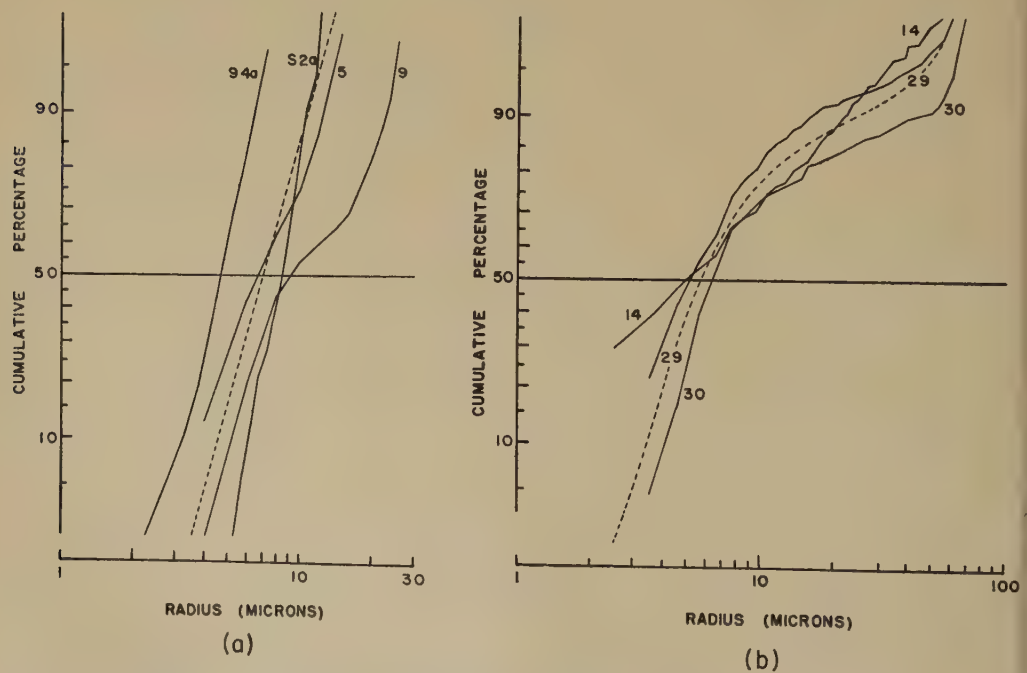

Figure 2. Droplet size distributions on cumulative percentage log-normal charts. The ordinate is the percentage of droplets of radius less than the abscissa. (a) Fair-weather cumulus : 94a, 52a, Diem; 5, 9, aufm Kampe and Weickmann. (b) Heavy cumulus : Weickmann and aufm Kampe 14, 29, cumulonimbus; 30, cumulus congestus. The dashed curves are idealized distributions.

* Private communication from aufm Kampe and Weickmann, Evans Signal Laboratories, Belmar, N.J.

indicates the narrowness of the distributions and their straightness shows that they are good approximations to log-normal distributions. One of Weickmann and aufm Kampe's cases (Fig. 2 (a), No. 9) is an exception. The plots for heavy cumulus (some of which are reproduced in Fig. 2 (b)) show much broader, and in some cases, bimodal distributions, that is, distributions made up of two log-normal distributions with different median radii. Bimodality appears also to be a property of specimen No. 9 : in view of the other anomalous properties of this specimen already noted, this justifies its exclusion from the fair-weather category.

The median radii of the remaining specimens vary from 4·5 to 12 microns, and their standard geometric deviations σ_g from 1·2 to 1·9. For purposes of computation, an idealized distribution for fair-weather cumulus was produced by taking a representative median radius of 7 microns and $\sigma_g = 1{\cdot}4$ and drawing a straight line (shown dashed) on Fig. 2 (a). In algebraic terms, this becomes

$$n_r\,dr = \frac{700}{r}\exp\left\{-22\left(\log_{10}\frac{r}{7}\right)^2\right\}dr \quad . \quad . \quad . \quad (1)$$

where $n_r\,dr$ is the number of drops per cm³ whose radii lie between r and $r + dr$ microns. It is replotted as a number distribution in Fig. 3 (a).

Calculations made by Howell (1949) on the formation of clouds in ascending moist air result in distributions which can be fitted by log-normal curves, though with somewhat smaller standard deviations than the measured ones. It is noteworthy that Palmer (1949) found that the distribution of rain-drops and snowflake sizes could also be fitted by a log-normal curve, and the values of σ_g he obtained were close to 1·4, but the significance

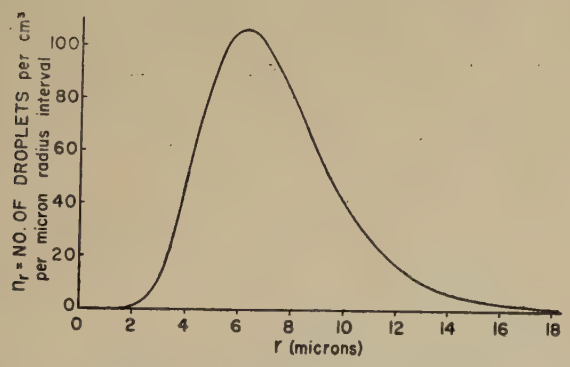

Figure 3 (a). Idealized fair-weather cumulus distribution. Droplet number versus radius.

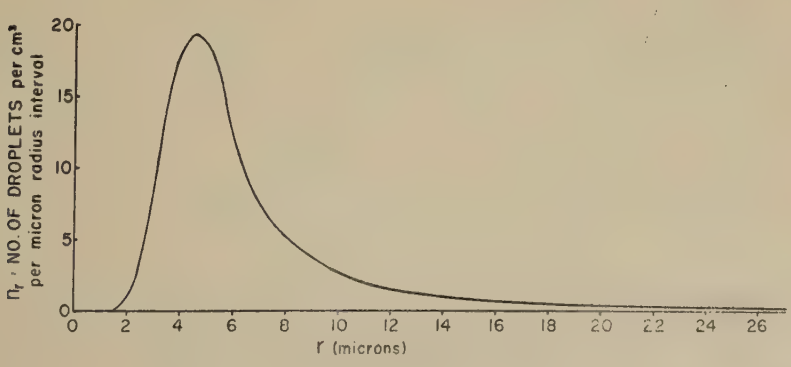

Figure 3 (b). Idealized heavy cumulus distribution. Droplet number versus radius.

of this finding is not immediately apparent. The important characteristic of the original data for these fair-weather cumulus clouds, which is well reproduced in the idealized curves, is the scarcity of drops of radii greater than 16 microns.

For purposes of the discussion in Section 4, an idealized heavy-cumulus distribution was drawn on Fig. 2 (b) as the dashed curve and converted to the number distribution of Fig. 3 (b). In converting from a percentage distribution, either the number of drops per cm^3 or the water content must be fixed arbitrarily. The number of drops was set at 600 per cm^3 for fair-weather (near the highest value found for this class) and 100 per cm^3 for heavy cumulus giving water contents of 1·5 g m^{-3} and 6 g m^{-3} respectively. These values are plotted on Fig. 1 as crosses.

It is not obvious from a glance at Figs. 3 (a) and 3 (b) why the idealized heavy-cumulus distribution should be called the broader of the two. Those droplets of radius greater than 20 microns, whose presence is an important characteristic, are not very numerous and do not stand out. This defect of presentation is removed in Fig. 4, where the two distributions are replotted as mass rather than number; the greater total water content of heavy cumulus, and the fact that it is mostly in the form of large droplets is very apparent. Indeed, the display errs in the other direction : between 50 and 60 microns the curve becomes extremely sensitive to minor details of behaviour of the upper end of the cumulative curve of Fig. 2 (b).

Cumulus clouds differ from layer clouds in having localized updraughts. A freshly-formed cumulus will have just a single central upward current of air with a velocity of a few metres per second. This updraught is maintained by the release of thermal energy by the condensing water vapour, and is prevented from accelerating indefinitely by entrainment of outside air, which makes the flow turbulent. Bowen (1950) has pointed out the importance of the updraught in prolonging the time that a raindrop spends in the cloud in the later stages of growth. In the following sections we shall discuss the action of the turbulence associated with the updraught in bringing about the earlier stages of growth.

3. Motion and collision of drops

(a) *Motion of droplets in air*

A sphere of radius r and mass m moves with constant velocity v_r through a fluid of density ρ and viscosity η. The drag force

$$D = \pi r^2 \tfrac{1}{2} \rho v_r^2 C_D, \quad . \quad . \quad . \quad . \quad (2)$$

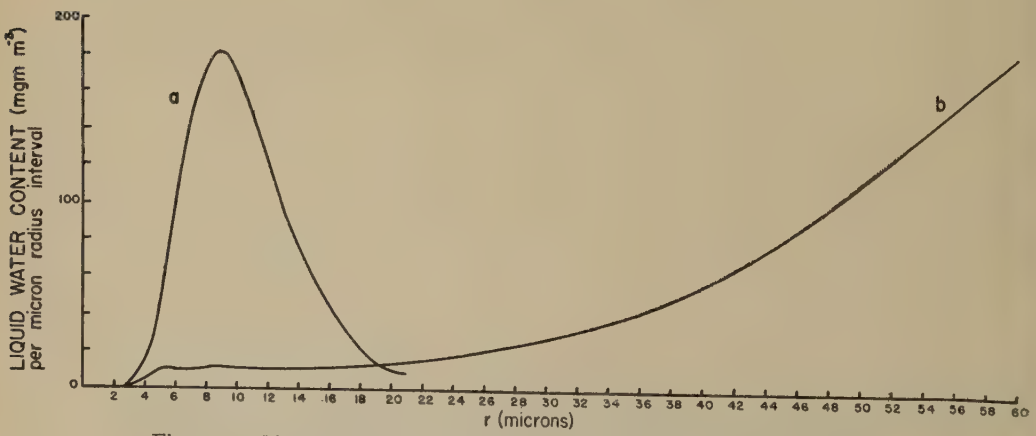

Figure 4. Idealized distributions replotted as water content versus radius. Left : fair-weather. Right : heavy cumulus.

where C_D is the drag coefficient and is a function of the Reynolds number

$$R = 2r\, v_r\, \rho/\eta$$

For values of $R \ll 1$ the relation between C_D and R simplifies to

$$C_D = 24/R$$

giving Stokes's Law

$$D = 6\pi r \eta v_r \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Cloud droplets of less than 30 microns radius, falling at their terminal velocity, obey Stokes's Law sufficiently closely for the purposes of the calculations which follow. Beyond this radius, an empirical law is used to take account of the departure. The viscosity of the air, η, is taken to be $1{\cdot}72 \times 10^{-4}$ g cm^{-1} sec^{-1}, the value at 0°C; it is independent of pressure. Where Stokes's Law is not obeyed, fall velocities depend on pressure as well as temperature; however, Langmuir (1948) found by calculation that even for a radius of 1 mm the terminal velocity is only 29 per cent higher at 8,000 m altitude than at 2,000 m, taking both pressure and temperature into account, and for smaller drops the difference in velocities is less and has been neglected.

When drag is the only force acting on the drop, and Stokes's Law applies, the equation of motion in one dimension is

$$m \frac{dv_r}{dt} = 6\pi r \eta\, (V - v_r), \quad . \quad . \quad . \quad . \quad (4)$$

where v_r is the velocity of the drop relative to stationary axes and V the air velocity. It is useful to define a ' time constant '

$$\tau_r = \frac{m}{6\pi r \eta} = \frac{2r^2 \rho_s}{9\eta} \quad . \quad . \quad . \quad . \quad (5)$$

where ρ_s is the density of the drop.
The equation of motion becomes

$$\tau_r \frac{dv_r}{dt} = V - v_r \quad . \quad . \quad . \quad . \quad . \quad (6)$$

(b) *Langmuir's equation*

Langmuir (1948) considered the problem of a current of air containing very small droplets, flowing past a fixed sphere. If the droplets continued in the same straight line, many would collide with the sphere, the number being proportional to the cross-section of the sphere. The collision cross-section would be πs^2, where s is the radius of the sphere. However, the air flows round the sphere in streamlines, and if the droplets showed no inertia they would be carried round the sphere and never collide with it. Inertia makes some collisions take place, but not as many as with perfectly straight paths. Using a differential analyser, Langmuir computed the paths of the centres of gravity of the droplets to find which would meet the sphere. He defined the ' Collision Efficiency ' E as the ratio of actual collision cross-section to πs^2. The results are contained in the equations

$$E = 0, \qquad K \leqslant 1{\cdot}214,$$

and

$$E = \left[1 + \frac{\frac{3}{4} \ln (2K)}{K - 1{\cdot}214}\right]^{-2}, \; K > 1{\cdot}214, \quad . \quad . \quad . \quad (7)$$

where K is a dimensionless quantity given by

$$K = \tau_r \frac{U}{s} = \frac{2r^2 \rho_s U}{9\eta s} = \frac{U}{U_0} \quad . \quad . \quad . \quad . \quad (8)$$

Here U is the velocity of the air (and of the droplets before encountering the sphere) relative to the sphere, and

$$U_0 = \frac{9\eta s}{2r^2 \rho_s}.$$

The form of Eq. (7) is shown in Fig. 5 where E is plotted against K on a log scale. The main feature is that below a critical value of K no collisions take place at all. This means that for given radii s and r, the relative velocity must be greater than some critical value for collisions to be possible.

Langmuir carried out calculations for a sphere much larger than the droplet, so that $s \gg r$. In the calculations which follow, the radii may be nearly equal in the early stages (so that application of his results is less reliable) but for convenience the term ' drop ' will be used for the larger particle and ' droplet ' for the smaller. The disparity in sizes allowed Langmuir to represent the droplet by a point mass at its centre of gravity. His values of E, therefore, are lower limits, since a droplet whose centre of gravity would miss the drop by less than r (the droplet radius) will actually collide with the drop. This will be allowed for, to some extent, by taking the capture cross-section for the drop and droplet to be $E\pi (s + r)^2$ instead of $E\pi s^2$. The number of droplets of radii r to $r + dr$ swept up per second by a drop of radius s is $U_0 KE\pi (s + r)^2 n_r dr$. The function KE is plotted against K on linear scales in Fig. 10 (curve a) and shows again most clearly that the rate of sweeping up droplets is zero until K reaches the critical value 1·214, and then increases rapidly.

A further assumption, which needs examining, is that the droplet is stationary relative to the air before it comes into the region of the drop. In the steady gravitational field, both drop and droplet move relative to the air, the droplet more slowly. If the air has just been accelerated, neither particle is stationary relative to it. In such cases U will be put equal to the relative velocity of drop and droplet regardless of the motion of the air. The errors arising from this assumption are not expected to be important.

The question remains whether a droplet which touches the drop always combines with it to form a single larger drop, or whether it sometimes ' bounces off.' Gabilly (1949) observed, under a microscope, coalescence between a captive drop and small droplets blown onto it. Dessens (1950) found by microscopic observations that if droplets are brought into contact, they always coalesce provided the radii are greater than about 1 or 2 microns. Gunn and Hitschfeld (1951) performed quantitative experiments which

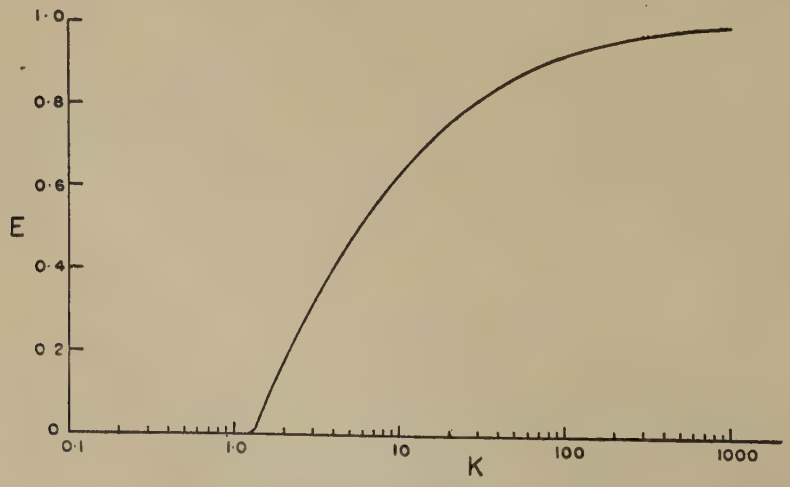

Figure 5. Langmuir's equation. Collision efficiency E versus relative velocity parameter K, Eqs. (7) and (8).

appear to show that a true collision always, or nearly always, results in coalescence, so that collection efficiency and collision efficiency will be assumed to have the same value, although in their experiments the radii of the drops were about 100 times the droplet radii.

4. Coalescence by gravity

(a) *Condition for collisions*

Consider a drop of radius s and a droplet of radius r in still air in a gravitational field g. If the locale is the earth's troposphere, g will have a value which varies little and is about 980 cm sec^{-2}. By equating the weight of the droplet to the drag force as given by Eq. (3) we find that the droplet will fall with velocity

$$v_r = g\frac{2r^2\rho_s}{9\eta} = g\tau_r$$

and the drop will fall with the greater velocity

$$v_s = g\frac{2s^2\rho_s}{9\eta} = g\tau_s$$

so that their relative velocity

$$U = \frac{2}{9\eta}\rho_s g(s^2 - r^2) = g(\tau_s - \tau_r) \quad . \quad . \quad . \quad (9)$$

and

$$K = \frac{U}{s}\tau_r = \frac{4\rho_s{}^2}{81\eta^2} g \frac{r^2}{s}(s^2 - r^2)$$

$$= 1{\cdot}64 \times 10^{-3}\frac{r^2}{s}(s^2 - r^2) \quad . \quad . \quad . \quad (10)$$

where r, s are in microns and $g = 980$ cm sec^{-2}.

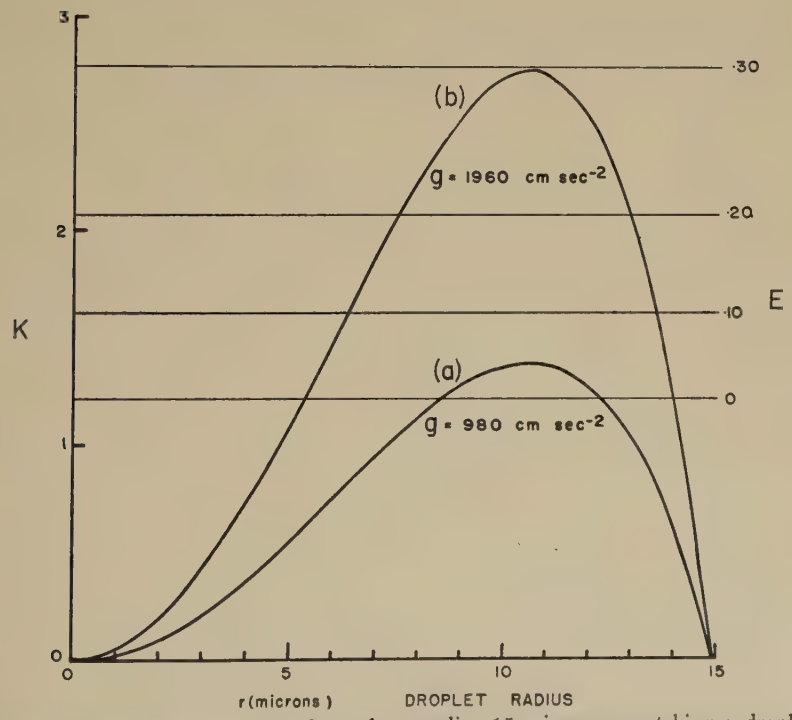

Figure 6. Relative velocity parameter K for a drop, radius 15 microns, overtaking a droplet, radius r. Gravitational field (a) 980 cm sec^{-2} (b) 1,960 cm sec^{-2}. Horizontal lines show values of K required for collision efficiency to reach 0·0, 0·1, 0·2 and 0·3.

In Fig. 6, K is plotted against r, the droplet radius, for a drop radius $s = 15$ microns. The lower curve is for $g = 980$ cm sec^{-2}. Lines are drawn at $K = 1{\cdot}214$, below which collisions are not possible, and at $K = 1{\cdot}62$, $2{\cdot}08$ and $2{\cdot}77$ for which E is 0·1, 0·2 and 0·3. Starting at the origin and following the lower curve, for small droplets the value of K is very small and they easily avoid the falling drop. As the droplet radius is increased K increases until it passes the critical value and collisions take place. At this stage, however, the droplet itself is falling with appreciable velocity, and further increases of droplet radius reduce the relative velocity between drop and droplet; K soon begins to decrease again. As the droplet approaches the drop in size, the relative velocity becomes zero, and so does K.

For a gravitational field of 980 cm sec^{-2}, collisions are only possible to the 15-micron drop over a limited range of radii of the droplets, and in this range the collection efficiency is small. At $r = 10{\cdot}6$ microns, K reaches a maximum of 1·38, but the efficiency is only 0·032.

(b) *Growth of a drop*

A calculation was made of the growth of a drop whose radius s is 15 microns, falling in a cloud which has the idealized characteristic of Eq. (1) (Fig. 3 (a)). The method of calculation was purely numerical. The drop grows by absorbing any droplets with which it collides; it is found to increase in radius by only $0{\cdot}42 \times 10^{-3}$ microns/sec, so that it would take roughly 40 min to increase from 15 to 16 microns radius. Even 18-micron drops (which are scarce in the type of cloud considered) only grow at the rate of 3×10^{-3} microns/sec.

The observed stability of fair-weather cumulus clouds is explained by these findings. It is evident that the value of g found on the earth is too small to coagulate them during their probable lifetime.

Differentiation of Eq. (10) with respect to r shows that for a given value of s, K has its maximum value at $r = s/2^{\frac{1}{2}}$. Substituting for r, Eq. (10) becomes

$$K_{\max} = 4{\cdot}1 \times 10^{-4}\, s^3 \quad . \quad . \quad . \quad . \quad (11)$$

A few representative values are given in Table 1. Evidently the collision efficiency changes very rapidly with s in the region of 15-microns radius, so that a modest shift in the drop-size distribution in the direction of larger radii would make a cloud unstable. Howell's (1949) results show that continued lifting of the cloud may bring about such a shift. However, according to one of his curves it would take over an hour for the largest drop to increase from 15 to 18 microns radius. Thus, although this is a factor in coalescence, anything that acts more quickly will be important.

TABLE 1

Drop radius s (microns)	$K_{\max}$	$E_{\max}$
14·3	1·214	0·0
15	1·38	0·032
18	2·39	0·25
30	11·1	0·66
100	410	0·98

With a gravitational field twice as great, K is doubled (Eq. 10), and curve (b) in Fig. 6 shows that collisions are possible for a wide range of droplet sizes and that the efficiency is about 0·3 at the maximum of K. Calculations of drop growth were carried out for $g = 1{,}960$ cm sec^{-2}; much faster growth was found. The results are shown in

Table 2. Doubling the gravitational field has increased the rate of growth of a 15-micron drop by about 13 times, and of an 18-micron drop by about 5 times. The relative velocities have all been doubled, so that the drop falls past twice as many droplets in a given time, but more important still, this has resulted in greatly increased collision efficiencies so that overtaking results in collision much more often.

TABLE 2

g (cm sec^{-2})	ds/dt (microns/sec)	
	$s = 15$ microns	$s = 18$ microns
980	$0{\cdot}42 \times 10^{-3}$	$3{\cdot}0 \times 10^{-3}$
1960	$5{\cdot}6 \times 10^{-3}$	$15{\cdot}1 \times 10^{-3}$

To pursue the growth of a drop, calculations were continued by a partly analytical method, in which the function E was approximated by suitably chosen fixed values, and the rate of growth ds/dt found by integration. The resulting graph of ds/dt against s was integrated numerically to obtain s against t. The overall accuracy is estimated to be about 10 per cent in rate of growth. At about $s = 30$ microns the drop becomes too big for Stokes's Law to apply, so a semi-empirical law for terminal velocity was used. At a later stage, the efficiency E can be taken to be about unity for all significant drop sizes.

Fig. 7 shows the way that the radius of the drop increases with time, starting from 20-microns radius (dotted curve). Changes in the droplet-size distribution of water content of the cloud through which the drop falls were neglected. Such changes are discussed below, but are too slow to affect the rate of growth seriously during the 12 min it takes for the drop to grow to 100-microns radius. At this stage, collisions between the drops themselves (which were also neglected in obtaining the curve) will become an important factor, and they turn into rain at an even faster rate than is shown in the figure.

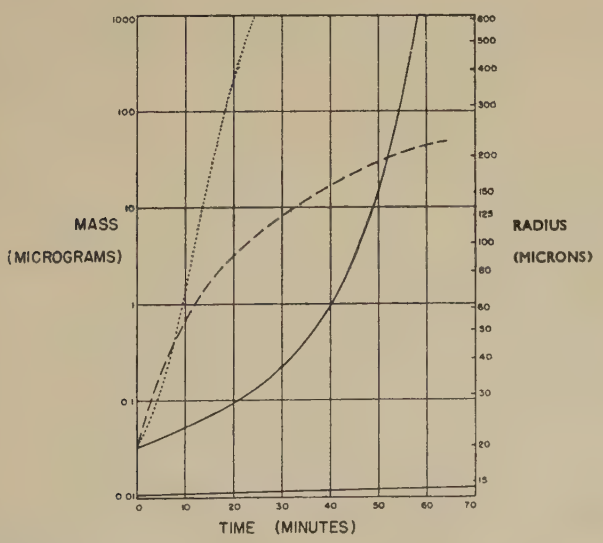

Figure 7. Growth of water drops by coalescence, and of ice particle by sublimation, in clouds. Solid lines, water drops, normal gravitational field; dashed curve, ice particle; from Houghton (1950) Fig. 5. Dotted line, drop with 20-micron radius in idealized fair-weather cumulus with gravitational field of 1,960 cm sec^{-2}.

The other curves are copied from Houghton (1950) and are for $g = 980$ cm sec^{-2}. The lowest line is for a rather homogeneous cloud of median radius 7 microns and water content 1 g m^{-3}; the other solid line is for a nimbostratus of water content 2 g m^{-3}. In both, growth is slow to start with. The dashed curve is for an ice crystal plate growing by sublimation in water-saturated air at -5°C. The initial growth is comparable with that of a water drop by coalescence for $g = 1{,}960$ cm sec^{-2}.

(c) *Change in the character of the cloud*

As part of the calculations of the effect of a doubled gravitational field, the number of droplets of each size absorbed in collisions by any drops large enough to do this in the first 1,000 sec was computed, and from this a new drop-size distribution was drawn (Fig. 8, curve (b)). Comparing curves (a) and (b), drop growth has been achieved by absorbing droplets between 5 and 14 microns radius almost exclusively. However, a size class can lose droplets not only by collisions with larger drops, but also by the growth of its own droplets to the next size class. When the effect of size change is taken into account, curve (b) becomes curve (c). The population is depleted further between 11 and 14 microns, and enhanced beyond 16-microns radius. The long ' tail,' characteristic of the broader cumulonimbus and cumulus congestus distributions, has begun to form.

To show a later stage in the life history of a cumulus, the idealized heavy cumulus described in Section 2 (and shown in Fig. 3 (b)) was replotted as curve (d) in Fig. 8 to the same scale as the other curves. The appearance of this curve in comparison with the others admits of the following interpretation in terms of a coalescence process. Though most of the drops have disappeared in the coalescence process, either by being absorbed or by growing into a bigger size class, the smaller drops have mostly escaped this fate. They have low collision efficiencies at any stage of the process. All the water that is lost between 4 and 12 microns appears mostly from 20 microns onwards in the form of a long ' tail ' to the curve. It is noticeable that even in the active stages of a coalescence process the ' tail ' of the distribution is very thinly populated, both because it takes a comparatively large amount of water to make one drop at these sizes, and because the drop grows quite rapidly, so that the chance of finding it in a given class is very small. Coalescence as such cannot change water content and so cannot directly account for the substantial increase from fair-weather to heavy cumulus that is evident in Fig. 1. It

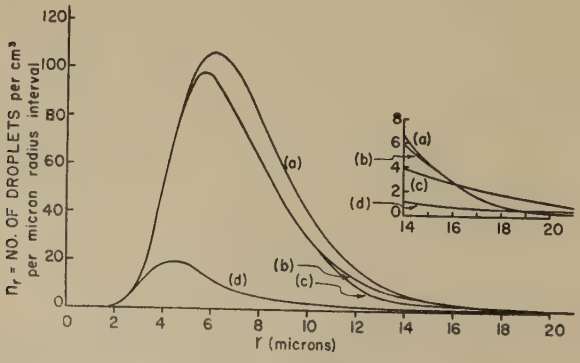

Figure 8. Drop-size distributions showing effect of gravitational field of 1,960 cm sec^{-2} acting for 1,000 sec. (a) Idealized fair-weather cumulus before field acts; (b) the same, with droplets removed by collision, but redistribution due to change of size not shown; (c) as (b) but drops redistributed according to growth; (d) idealized stormy cumulus for comparison. The last portion of each curve is redrawn with expanded vertical scale.

could be simply caused by continued condensation from the rising moist air, so that the number of small droplets reaches an equilibrium, with the larger drops removing them in collisions, and condensation replenishing the supply. But it may also be, at least in part, that the heavier fraction of drops in an upper-air region fell to the levels where the measurements were made and brought about a concentration of water. Possibly the method of determining water content has some systematic error which depends on the shape of the distribution.

The gravitational field used for this calculation is not found in the earth's atmosphere, of course. However, a steady acceleration of air in which a drop and a droplet find themselves would produce the same effect as a gravitational field, and could bring about collisions. Any kind of acceleration of the air would produce effects similar to those of a gravitational field, and could also cause collisions. In the next section we shall consider systematic and random motion of the air in a straight line, as intermediate stages in the approach to the actual motion of turbulent air.

5. One-dimensional motion

Inside a cumulus cloud the air is in turbulent motion so that local accelerations are always taking place. The problem is to find out how effective they are in bringing about collisions. A simple approach is used at first to gain insight into the physics and mathematics. In this section the one-dimensional problem only is considered, and to start with, the random motion of actual turbulence is replaced by a sinusoidal motion, so that well-known methods of solution can be used.

(a) *Sinusoidal applied motion*

Consider a parcel of air containing the drop (radius s, time constant τ_s) and the droplet (radius r, time constant τ_r) to be in sinusoidal motion with peak velocity $\bar{V}$ and angular frequency ω. Then the instantaneous air velocity

$$V = \mathrm{Re}\,[\bar{V} \exp(i\omega t)]$$

The motion of the droplet is given by substituting for V in Eq. (6), and solving for v_r. The droplet velocity is

$$v_r = \mathrm{Re}\left[\frac{\bar{V}}{1 + i\omega\tau_r} \exp(i\omega t)\right]$$

or in the notation customary in electrical problems, where the symbols Re and exp ($i\omega t$) are omitted

$$v_r = \frac{V}{1 + i\omega\tau_r} \qquad (12)$$

For the drop

$$v_s = \frac{V}{1 + i\omega\tau_s} \qquad (13)$$

so the relative velocity

$$U = \frac{V\, i\omega\,(\tau_s - \tau_r)}{(1 + i\omega\tau_s)\,(1 + i\omega\tau_r)} \qquad (14)$$

Taking absolute values only, the relative velocity of the drops (compared to the air velocity) is given by

$$\left|\frac{U}{\bar{V}}\right|^2 = \frac{\omega^2\,(\tau_s - \tau_r)^2}{(1 + \omega^2\,\tau_s^2)\,(1 + \omega^2\,\tau_r^2)} \qquad (15)$$

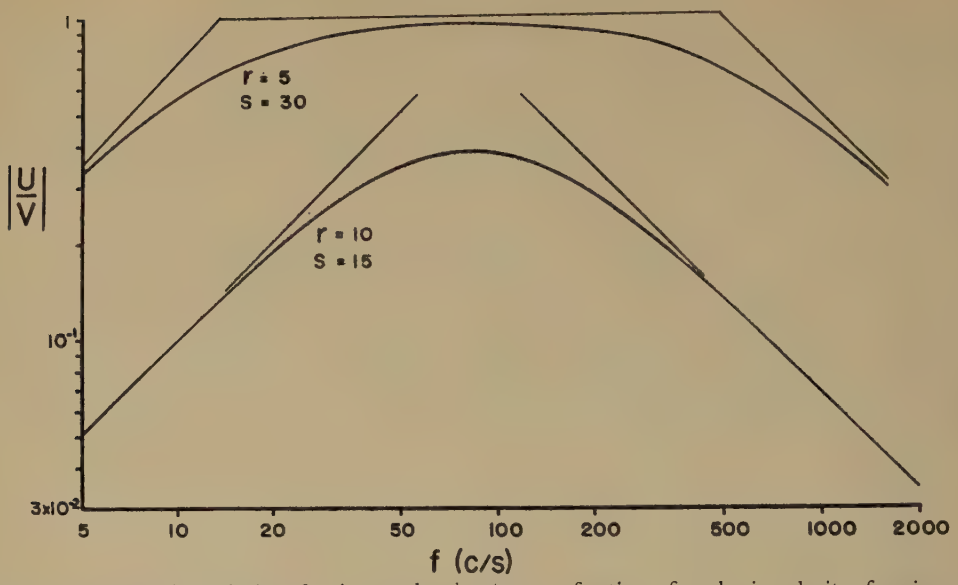

Figure 9. Peak relative velocity of a drop and a droplet as a fraction of peak air velocity, for sinusoidal motion of frequency f. Upper curve : radii 30 and 5 microns. Lower curve : radii 15 and 10 microns.

This function is shown in Fig. 9, where $|U/V|$ is plotted against $f = \omega/2\pi$ for $r = 5$ microns and $s = 30$ microns (upper curve). At very low frequencies both drop and droplet move with the air and there is very little relative velocity; what little there is arises from the lag of the drop and is proportional to ω. This lag increases until above about 30 c/s the drop stays almost still; the droplet still moves with the air and so the relative velocity almost equals the air velocity. At about 200 c/s, however, the droplet also begins to lag appreciably, and at higher frequencies still, it moves less and less, so that the relative velocity falls off as ω^{-1}. The asymptotic behaviour towards zero and infinite frequencies is indicated by the straight lines. The lower curve is for radii of 10 and 15 microns. Since the droplet and drop are similar in size, they move more nearly together, the relative velocity at any given frequency is much less than before (it never exceeds two-fifths of the air velocity in this case) and the level portion of the curve is missing.

Now consider some fixed frequency, and a fixed drop radius s, and vary the droplet radius r. The parameter K in Langmuir's equation is proportional to τ_r so that for very small droplets where τ_r is very small, K is well below the critical value and collisions are impossible. On the other hand, if the droplet is nearly as big as the drop, τ_r nearly equals τ_s, and Eq. (15) shows that the relative velocity U becomes very small. At intermediate values of τ_r, K can exceed the critical value for moderate values of V, and collisions take place. This is just the kind of behaviour found earlier for gravitational excitation and shown in Fig. 6.

The similarity between the effects of a fluctuating air motion and gravity becomes an analogy when the period is long compared with the time constant of the drop. Then $\omega\tau_s \ll 1$ and Eq. (15) becomes

$$U^2 = V^2\,\omega^2\,(\tau_s - \tau_r)^2\,[1 - \omega^2\,(\tau_s + \tau_r)^2 + \ldots] \quad . \quad . \quad (16)$$

Putting $|V|\,\omega = A$, the peak acceleration of the air, then the peak relative velocity

$$U \simeq A\,(\tau_s - \tau_r) \quad . \quad . \quad . \quad . \quad (17)$$

closely parallel to Eq. (9), but with the peak acceleration A replacing the gravitational field g.

(b) *Turbulence*

Turbulence is caused by wind shear such as that which results from the action of the ground on the wind. In a vertically developed cloud there is a shear between the updraughts and the surrounding air. Entrainment takes place and the motion becomes strongly turbulent.

Although turbulence has been studied theoretically and experimentally for several decades, present-day knowledge is still incomplete. Much work has gone into developing an adequate mathematical description but this is still in an active stage of development. Experiments ' in the field ' have mostly been aimed towards learning about diffusion of water vapour or smoke in the clear atmosphere, and none seems to have been carried out inside clouds. Many flights have been made, it is true, in which the accelerations (or ' bumps ') of the aircraft are measured, but the aircraft travels so far in a short time that this measurement merely reveals the distribution of updraughts along a horizontal line, rather than the variation of the velocity of one region with time. The most useful results for the present discussion are those obtained in wind tunnels.

In describing turbulent flow, any steady motion of the air must first be subtracted out so that the mean air velocity taken over a period of time tends to zero as the period tends to infinity. The work of Simmons and Salter (1934), Townend (1934), Taylor (1935) and Townsend (1947, 1950, 1951) shows that turbulence in air well clear of any boundaries, such as the walls of the wind tunnel (or, in the atmosphere, the ground), can then be expected to have the following properties. The motion of the air is *random* and the distribution of velocity in any given direction at a point is a Gaussian or normal error curve. The motion is also *stationary*, *homogeneous* and *isotropic*, which means that the statistical properties of the motion are independent of time, place, and the direction of the axes of reference.

The velocity of a parcel of air which contains a drop and some droplets can be resolved along one axis, and the occurrence of collisions may be calculated in one dimension. Since the turbulence is isotropic, the statistical situation must be the same in each of the three dimensions. However, the components of motion in orthogonal directions are not necessarily independent. Just how the one-dimensional results can be used to give the rate of occurrence of collisions in a three-dimensional system, and to what extent, will be discussed in Section 6. Of course, if the motion were really entirely in one direction, a drop would make all possible collisions in a very few excursions through the droplets, and would then move in a clear channel, making no more collisions. Any transverse motion, however, would ensure that the drop was always encountering a fresh sample of droplets.

(c) *Random applied motion*

The problem of coalescence by one-dimensional excitation in general can be approached in the following way. The air velocity V is a function of time and can be written $V(t)$; similarly, the relative velocity is written $U(t)$. They are linked by two linear differential equations,

$$v_r + \tau_r \frac{dv_r}{dt} = V(t) \quad . \quad . \quad . \quad . \quad (18)$$

$$v_s + \tau_s \frac{dv_s}{dt} = V(t) \quad . \quad . \quad . \quad . \quad (19)$$

and the linear equation

$$U(t) = v_r - v_s \quad . \quad . \quad . \quad . \quad (20)$$

so that $V(t)$ can be regarded as the cause and $U(t)$ the effect in a linear relationship. For

calculating the rate of collisions, it is necessary to know the probability distribution of the relative velocity, and nothing more. Since the probability distribution is known to be Gaussian, it is completely specified by its mean square value. The problem reduces to one of finding the mean square relative velocity σ_U^2 in terms of the air velocity $V(t)$.

The writers were fortunate in having the collaboration of Z. A. Melzak, who solved this problem using a method believed to be due originally to N. Wiener. The air velocity $V(t)$ is a random function with stationary properties, so that it can be described by its mean square σ_V^2 and by its spectrum. The spectrum is used in the form of a spectral density function $G(\omega)$ (as defined, for example, by Wang and Uhlenbeck (1945)). It is normalized to make

$$\sigma_V^2 = \int_0^\infty G(\omega)\, d\omega \qquad . \qquad . \qquad . \qquad . \qquad (21)$$

The mean square relative velocity is then given by

$$\sigma_U^2 = \int_0^\infty |U/V|^2\, G(\omega)\, d\omega \qquad . \qquad . \qquad . \qquad (22)$$

Thus σ_U^2 is a weighted mean square of the air velocity spectrum, where σ_V^2 was the unweighted mean square. The weighting factor is the sinusoidal response given by Eq. (15). It can be seen that, for a given spectrum of the air velocity V, σ_U^2 is proportional to σ_V^2, the constant of proportionality depending on τ_s and τ_r. It can also be seen that a spectrum that is concentrated around 100 c/s, when weighted by the square of the frequency response shown in the lower curve of Fig. 9 would give a larger value of σ_U^2 than one whose spectral density was small around 100 c/s. Thus, although one does not wish to know anything about the spectrum of U, something must be known about the spectrum of V for the method to give any value at all for σ_U^2.

The integral (22) was evaluated by Melzak for a number of trial spectra of the air velocity V. The trial spectra are listed in Table 3 in the first two columns. In the expressions for $G(\omega)$, ω_0, ω_H, ω_L are fixed frequencies characteristic of the particular spectrum in which they appear. The resulting formulae for σ_U^2 were expanded in ascending powers of ω_0 (or ω_H and ω_L) and the first two terms are shown in the third column of Table 3. These expressions show the dependence of the mean square relative velocity on the drop and droplet time-constants as well as on the shape and characteristic frequency of the spectrum.

TABLE 3.

Name	Air-velocity spectrum $G(\omega)$	Mean square relative velocity σ_U^2	Mean square acceleration of air σ_A^2
Markoff (exponential correlation function)	$\sigma_V^2 \frac{2\omega_0}{\pi} \frac{1}{\omega^2+\omega_0^2}$	$\sigma_V^2 \omega_0 \frac{(\tau_s-\tau_r)^2}{\tau_s+\tau_r} \times[1-\omega_0(\tau_s+\tau_r)+\ldots]$	Does not converge
Gaussian (normal error curve)	$\sigma_V^2 \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \omega_0^{-1} \exp\left(-\frac{\omega^2}{2\omega_0^2}\right)$	$2\sigma_V^2\omega_0^2(\tau_s-\tau_r)^2 \times[1-3\omega_0^2(\tau_s^2+\tau_r^2)\ldots]$	$2\sigma_V^2\omega_0^2$
Uniform band ω_L to ω_H	$\begin{cases} 0 & \omega<\omega_L \\ \frac{\sigma_V^2}{\omega_H-\omega_L} & \omega_L\leqslant\omega\leqslant\omega_H \\ 0 & \omega_H<\omega \end{cases}$	$\sigma_V^2\{\omega_H\omega_L-\frac{1}{3}(\omega_H-\omega_L)^2\}(\tau_s-\tau_r)^2 \times[1-\omega_H\omega_L(\tau_s^2+\tau_r^2)+\ldots]$	$\sigma_V^2\{\omega_H\omega_L-\frac{1}{3}(\omega_H-\omega_L)^2\}$
Delta function (sinusoidal oscillation)	$\sigma_V^2\,\delta(\omega-\omega_0)$	$\sigma_V^2\omega_0^2(\tau_s-\tau_r)^2 \times[1-\omega_0^2(\tau_s^2+\tau_r^2)+\ldots]$	$\sigma_V^2\omega_0^2$

If the characteristic frequency ω_0 is small enough, all terms except the first can be neglected. More specifically, the condition is

$$\omega_0 \tau_s \ll 1 \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$(\text{or } \omega_H \tau_s \ll 1) \quad . \quad . \quad . \quad . \quad . \quad (23')$$

In all cases except the first, the expressions are very similar, involving $(\tau_s - \tau_r)^2$ and a quantity having the dimensions of mean square acceleration. For comparison, the mean square acceleration of the air motion

$$\sigma_A^2 = \int_0^\infty \omega^2 G(\omega)\, d\omega \quad . \quad . \quad . \quad . \quad (24)$$

has been calculated for each spectrum and is listed in the last column.

It is evident that when condition (23) is fulfilled, in any case except the first, the mean square velocity and the mean square air acceleration are related by the equation

$$\sigma_U^2 = \sigma_A^2 (\tau_s - \tau_r)^2 \quad . \quad . \quad . \quad . \quad (25)$$

The analogy with Eq. (9)

$$U = g(\tau_s - \tau_r)$$

for a gravitational field is complete.

Eq. (25) applies to the uniform band spectrum when $\omega_H \tau_s \ll 1$ and since any spectrum can be split up into a large number of approximately uniform bands, it must apply (in the limit) to any spectrum which satisfies this condition in each band. It does not apply if σ_A^2 receives an appreciable contribution from those parts of the spectrum for which $\omega\tau_s$ is near to or greater than unity. The Markoff spectrum is of this type, and its formula for σ_U^2 differs considerably from the others; actually the integral for σ_A^2 does not converge.

Until more is known about the spectrum of turbulence inside clouds, it is not possible to say whether condition (23) will be met in practice. If measurements show that it is met, then the simple relationship (25) will hold, and only the quantity σ_A^2 need be used. If the condition is not met, then equation (22) will have to be applied.

(d) *Collisions by random applied motion*

There is one difference between a gravitational field and a random one-dimensional acceleration that has to be taken into account; it is that the relative velocity U is random, not steady as it was in the computations of Section 4. The probability of the relative velocity lying between U and $U + dU$ is

$$P(U)\, dU = \frac{1}{\sigma_U (2\pi)^{\frac{1}{2}}} \exp\left(-\frac{U^2}{2\sigma_U^2}\right) dU \quad . \quad . \quad . \quad (26)$$

The probability density $P(U)$ is normalized so that

$$\int_{-\infty}^{\infty} P(U)\, dU = 1.$$

The mean speed $\overline{|U|}$ is given by the weighted integral

$$\overline{|U|} = \int_{-\infty}^{\infty} |U|\, P(U)\, dU = 2\int_0^\infty U\, P(U)\, dU \quad . \quad . \quad (27)$$

since $P(U) = P(-U)$. The volume swept out by a drop, radius s, in unit time is $Q_1 = \pi s^2 \overline{|U|}$. If the collection efficiency were unity, the quantity Q_1 would give the number of droplets captured per second very simply. Since the collection efficiency is

not unity, Q_1 must be replaced by an equivalent swept volume per unit time Q by replacing πs^2 by $E\pi(s+r)^2$, as discussed in Section 3. Since E depends on U, it must be taken inside the integral. Then the equivalent volume swept per unit time

$$Q = \pi(s+r)^2 \, 2 \int_0^\infty E\,U\,P(U)\,dU \quad . \quad . \quad . \quad (28)$$

In the above formula as it stands, E depends not only on U, but also on s and r, so that the integral would have to be evaluated afresh for every combination of drop and droplet. This can be avoided by putting

$$U = U_0\,K, \qquad \sigma_U = U_0\,\sigma_K,$$

where

$$U_0 = \frac{9\eta s}{2r^2\,\rho_s},$$

K is the dimensionless parameter in Langmuir's equation given by (8) and σ_K is the root mean square of K. Then the probability distribution (26) becomes

$$P(U)\,dU = \frac{1}{\sigma_K(2\pi)^{\frac{1}{2}}} \exp\left(-\frac{K^2}{2\sigma_K{}^2}\right) dK = P(K)\,dK \quad . \quad . \quad (29)$$

and the equivalent swept volume per unit time

$$\begin{aligned} Q &= \pi(s+r)^2\,2U_0 \int_0^\infty E(K)\,P(K)\,K\,dK \\ &= \pi(s+r)^2 \;\; U_0 \int_0^\infty E(K)\,\frac{K}{\sigma_K}\left(\frac{2}{\pi}\right)^{\frac{1}{2}} \exp\left(-\frac{K^2}{2\sigma_K{}^2}\right) dK \\ &= \pi(s+r)^2 \;\; U_0\,\alpha, \quad . \quad . \quad . \quad . \quad . \quad (30) \end{aligned}$$

where α stands for the definite integral. The value of α depends on σ_K but not otherwise on s and r.

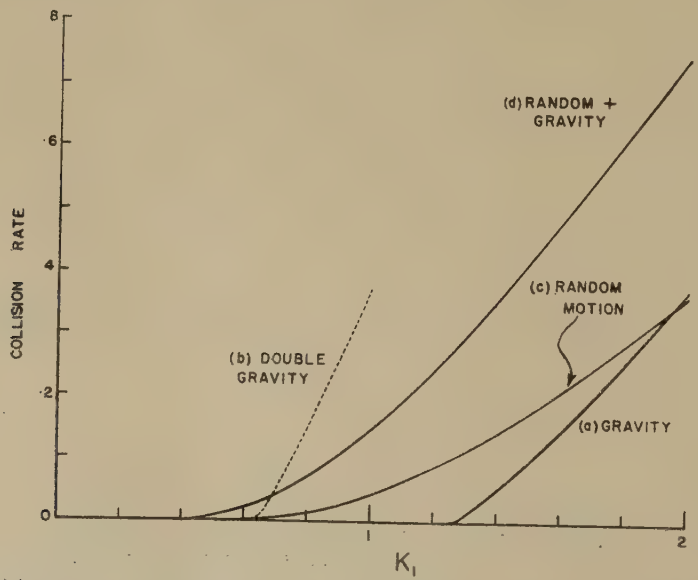

Figure 10. Collision rates. When the ordinate is multiplied by $U_0\,\pi(s+r)^2\,n_r\,dr$, it gives the number of collisions per second between the drop, radius s, and droplets of radius r to $r + dr$. The abscissa K_1 is a function of s and r. (a) Gravitational field 980 cm sec^{-2}; (b) Gravitational field 1,960 cm sec^{-2}; (c) Random relative velocity whose r.m.s. value equals that produced by field of 980 cm sec^{-2}. (d) as (c) with superimposed steady relative velocity produced by field of 980 cm sec^{-2}.

In Fig. 10 the collision rates under various conditions are plotted. The abscissa is chosen to make comparisons possible; it is

$$K_1 = \frac{980}{s} \tau_r (\tau_s - \tau_r) \quad . \quad . \quad . \quad . \quad (31)$$

which is a function of s and r only, with its absolute value chosen so that in the ordinary gravitational field $K = K_1$. Curve (a) shows the function KE introduced in Section 4. (In Fig. 6, curve (a) can be read as K_1 against r for $s = 15$ microns). With a doubled gravitational field $K = 2K_1$ and curve (b) results. In curve (c) the integral α is plotted with $\sigma_K = K_1$, which represents an intensity of turbulence such that the r.m.s. relative velocity equals the steady relative velocity attained with ordinary gravity (this would result from a r.m.s. acceleration of 980 cm sec^{-2} if condition (23) holds). The main trend of the curve of α versus σ_K is to increase the rate of sweeping up droplets with increase of σ_K. This is similar to the trend shown in curve (a), where collision rate increases with K. However, there is no critical value of σ_K below which α is zero : α is still finite, however small σ_K becomes. The physical interpretation of this difference in behaviour is, that even if the r.m.s. relative velocity σ_U is less than the critical value, U itself will exceed the critical value for some part of the time. For example, if $K = 1{\cdot}0$ with gravitational excitation, no collisions occur, because $K < 1{\cdot}214$; while if $\sigma_K = 1{\cdot}0$ with random excitation, collisions occur at the rate given by $\alpha = 0{\cdot}15$, which corresponds to a mean collection efficiency of 0·049, still quite a large value. On the other hand, the reduction of collision rate with decreasing σ_K is quite rapid; at $\sigma_K = 0{\cdot}6$, for example, the mean collection efficiency is only 0·007, so that collisions, although still not impossible, are much rarer.

In an actual cloud, gravity and turbulence will occur together. Horizontal motion is the result of turbulence only, but vertical motion is the result of gravity and turbulence combined. The probability distribution of the vertical component of velocity is given by

$$P(U)\,dU = \frac{1}{\sigma_U (2\pi)^{\frac{1}{2}}} \exp\left(-\frac{(U + U_1)^2}{2{\sigma_U}^2}\right) dU \quad . \quad . \quad . \quad (32)$$

where $U_1 = 980\,(\tau_s - \tau_r)$ is the steady part of the relative velocity due to gravity. The random vertical motion of the air alternately aids and opposes gravity in bringing about coalescence. The resulting collision rate

$$\begin{aligned} Q' &= \pi (s + r)^2 \int_{-\infty}^{\infty} \frac{EU}{\sigma_U (2\pi)^{\frac{1}{2}}} \exp\left(-\frac{(U + U_1)^2}{2{\sigma_U}^2}\right) dU \\ &= \pi (s + r)^2 U_0 \int_{-\infty}^{\infty} \frac{EK}{\sigma_K (2\pi)^{\frac{1}{2}}} \exp\left(-\frac{(K + K_1)^2}{2{\sigma_K}^2}\right) dK \\ &= \pi (s + r)^2 U_0 \alpha' \quad . \quad . \quad . \quad . \quad . \quad (33) \end{aligned}$$

where $K_1 = U_1/U_0$, and α' is the value of the integral. Assuming for example that the r.m.s. value of the random part of the relative velocity equals the steady part of the relative velocity due to gravity, so that $\sigma_K = K_1$, the value of the integral α' is shown in Fig. 10 as curve (d). The effect of turbulence is apparently to move the curve bodily to the left from (a) to (d). This greatly assists the collision process in the early stages, where the drops are so small that K_1 is less than 1·214 for most of them. The rate of growth will be comparable with that for double gravity (curve (b)). Once the drop has grown somewhat, however, so that the larger values of K_1 apply, the effect of this amount of turbulence is intermediate between the effects of single and double gravity.

The position regarding one-dimensional excitation of a cloud can be summarized as follows : qualitatively, the effect is very much the same as gravity, except that the onset of coalescence is more gradual. Quantitatively, given any value for the mean square acceleration of the turbulent air, and provided a certain condition is fulfilled, the rate of growth of a drop in a given cloud can be computed. No measured value for mean square acceleration is available at the present time. If the condition is not fulfilled, the complete spectrum of the turbulence will be required.

6. Three-dimensional motion

(a) *Extension of one-dimensional results*

The derivation of a three-dimensional treatment of turbulence from the one-dimensional results of the previous section can proceed on the following lines. The relationship between the component of relative velocity U_x and the component of air acceleration A_x in one direction is linear and independent of the velocity and acceleration in the other two directions. If we consider an isotropic air motion, for which condition (23) is valid, the mean squares of the three components of relative velocity are related to the mean squares of the three components of air acceleration as follows :

$$\sigma_{Ux}^2 = \sigma_{Uy}^2 = \sigma_{Uz}^2 = \sigma_{Ax}^2 (\tau_s - \tau_r)^2 = \sigma_{Ay}^2 (\tau_s - \tau_r)^2 = \sigma_{Az}^2 (\tau_s - \tau_r)^2 \quad . \quad (34)$$

If W is the relative speed, its mean square

$$\sigma_W^2 = \sigma_{Ux}^2 + \sigma_{Uy}^2 + \sigma_{Uz}^2 = 3\sigma_{Ux}^2 \quad . \quad . \quad . \quad (35)$$

Temporarily, the assumption is made that the components of air motion in the three directions are mutually independent (as they are for molecular motion in the kinetic theory of gases); the components of relative velocity are then mutually independent also. In such a case, the well known Maxwellian law can be applied to the relative velocity, and the probability distribution of relative speed

$$P(W)\,dW = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{W^2}{\sigma_{Ux}^3} \exp\left(-\frac{W^2}{2\sigma_{Ux}^2}\right) dW \quad . \quad . \quad (36)$$

(See for example Jeans (1948), Section 91). The expression (36) is normalized.

The calculations can now proceed as in Section 5, but with the Eq. (26) replaced by the new probability distribution (36). The new distribution is narrower than the old, so the result of the procedure would be a curve of rate of collisions resembling Fig. 10 (c) in essence, but differing in detail. In particular, in one dimension appreciable values of α occur even at small values of σ_K because the relative velocity U exceeds its r.m.s. value by say 50 per cent for a considerable part of the time. With the narrower distribution of speed W in three dimensions, W exceeds its r.m.s. value by such a factor much less often, so that the collision rate would fall off very rapidly as σ_K went below unity. In fact, since the distribution of W is intermediate in width between the broad distribution of U in one-dimensional random motion and the infinitely narrow distribution of U for steady motion, the curve of rate of collision would be intermediate in shape between curves (c) and (a) of Fig. 10.

For these computations, it would be necessary to know the mean square acceleration of the three-dimensional turbulent motion, so that σ_W^2 can be calculated for each pair of drop and droplet sizes. As before, this quantity has not been measured.

Besides this practical difficulty, however, there is a fundamental objection to the method described. The assumption was made that the components of air motion were mutually independent; this is not true except on the molecular scale (Batchelor (1952)).

How this would affect the results is not easy to say. Instead of attempting to modify the procedure to allow for the interaction of velocities in three dimensions, it seems it would be more profitable to start with the more fundamental approach to the nature of turbulent motion that is now becoming available.

(b) *The modern theory of turbulence*

Batchelor (1952) gives a good account of the modern theory of turbulent motion. The velocity field at an instant of time is subjected to a three-dimensional Fourier transformation. The result is a spectrum in three-dimensional wave-number space. The distance of a point in wave-number space from the origin is the reciprocal of the size of the eddies which it represents, so that the nearest points belong to the largest eddies.

Heisenberg (1948) has developed the theory by using dimensional arguments. He arrives at a picture in which turbulence is an intermediate stage through which energy passes on its way from the source to eventual dissipation by viscous friction. Eddies of a particular size are always being converted by inertial forces into smaller eddies. The energy from the source goes at first into large eddies (of size comparable to the boundaries of the system), which then hand it down through smaller and smaller eddies. As a certain size is reached, the energy goes into viscous friction instead of into still smaller eddies, and the spectrum does not extend appreciably beyond this size.

This wave-number theory is not very closely related to the frequency-spectrum treatment considered previously, chiefly because the wave-number spectrum describes the state of a large region of space at an instant of time, while the frequency spectrum describes the movements in time and space of a single element of fluid. However, in a semi-quantitative way one can associate the small eddies with high frequencies, and it is possible, at least in principle, to transform equations from one system to another.

The virtue of the new theory is that its results are expressed in terms of very few initial facts. Instead of a mean square velocity or acceleration and a frequency spectrum, neither of which is known, the main quantity which determines the nature of the motion is the rate of supply of energy per unit mass ϵ. The energy is supplied by thermal processes in the atmosphere, and can be deduced from such facts as the temperature and humidity, which have been measured in typical cases. Braham (1952) has drawn up an energy budget for a thunderstorm, and a similar budget for a fair-weather cumulus cloud would give the required value of ϵ.

If the new theory of turbulence is to be applied to the problem of collision between the drops, it must provide the probability distribution of acceleration, $P(|A|)\,d|A|$. The relationship between $|U|$ and $|A|$ must then be re-established and $P(U)\,dU$ obtained. The work leading up to curves (c) and (d), Fig. 10, can then be repeated with the new distribution. Of course, an indirect determination of turbulent motion of this kind cannot replace reliable and accurate direct measurements, but until such measurements can be made, it perhaps offers a means of reaching a quantitative conclusion.

7. Conclusions

Observations from aircraft and by radar leave no doubt that rain can, and often does, form in cumulus clouds at temperatures higher than 0°C, so that ice cannot play any part in the mechanism. Coalescence is an alternative mechanism, and there is observational evidence of it. When a fair-weather cumulus changes to the heavy forms the number of droplets per unit volume decreases by about four times, clearly indicating that coalescence between drops is taking place.

Fair-weather cumulus clouds contain just a few drops of greater than 15-microns radius, none of 20 microns or greater. Calculations, based on Langmuir's consideration of collisions between drops, show :

(1) Growth by gravity is impossible for a drop of 14-microns radius, and very slow with radii between 15 and 20 microns. This accounts for the observed fact that fair-weather cumulus clouds do not automatically precipitate but often remain stable for long periods.

(2) A gravitational field twice as strong would make growth possible for drops down to 11·5 microns radius, and significant in the region 15 to 20 microns (the rate of growth of a 20 micron drop with doubled gravity is about the same as that of a 40 micron drop with single gravity). A comparable random acceleration would be just as effective.

(3) Any component of fluctuating acceleration parallel to gravity adds to the coalescence effect, and if this component is of the same order as the actual gravitational field, significant growth will occur in the 15 to 20 micron region. In turbulent air, fluctuating accelerations are always present.

Application of these studies calls for measurements of mean square acceleration inside the clouds and some check on the form of its spectrum. Rates of growth of drops could then be calculated (with reservations as to the way a three-dimensional motion such as turbulence is resolved into three one-dimensional motions). Instead of a one-dimensional treatment, it may be that an entirely theoretical approach through energy considerations would be more profitable.

Young cumuliform clouds are notably free from suitable growth drops, and are therefore particularly in need of some agency to start the precipitation process. Cumulus clouds, as a class, have the most turbulence, and this turbulence may supply their need. Heavy cumulus clouds, on the other hand, have a much broader droplet-size distribution with a long ' tail ' extending to large radii. It has been shown not only how turbulence could initiate rain, but that in doing so it would broaden the size distribution of the cloud in just this way.

If turbulence is the primary cause of rain which forms in warm cumulus clouds, it must also be an important factor in similar clouds which are cold enough for the ice phase to exist. Even where the Bergeron-Findeisen process is predominant, turbulence would help a falling snowflake to collect supercooled droplets. If coalescence by turbulence becomes established in the lower part of the cloud, it may affect an ice-phase process by reducing the supply of droplets. Indeed, if turbulence alone can account for precipitation in the comparatively few cases where the entire cloud is warmer than 0°C, there is no need to look elsewhere for the cause of showers from isolated clouds which lie entirely below the temperature of spontaneous freezing of droplets (apparently about — 12°C); such clouds are a large proportion of all shower clouds.

Acknowledgments

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Observations of rainfall from warm clouds

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SUMMARY

Rainfall from non-freezing clouds has been widely reported but detailed observations are sparse. During June 1952, ten days of observations were made during trade-wind weather conditions on the island of Oahu. Rainfall was observed during each of the 10 days, and in no instance did cloud temperatures below 7°C occur. Aircraft were used to obtain temperature and humidity soundings supplementary to the Honolulu radiosonde and to measure the dimensions of the clouds. Raindrop sizes were recorded on a continuously moving dye-treated paper tape at two locations. Data on rainfall were obtained from 5 recording raingauges on the crest of the Koolau Mountain Range and from 32 standard raingauges in an area of 130 mi^2. Pilot-balloon ascents were made daily on the windward coast of Oahu to supplement the Honolulu Weather Bureau wind data. A continuous record of cloud conditions and motions was made by lapse-time motion-picture photography and supplemented with photographs from aircraft.

Rainfall intensities exceeding 3 mm/hr were observed from clouds with bases at 2,000 ft (18·6°C) and tops at 7,500 ft (10·2°C). Raindrop sizes as large as 2 mm in diameter were observed from clouds 6,000 ft deep. Light-intensity rains from small cumulus clouds frequently consisted of raindrops which were relatively large (2 mm diameter) but low in number.

1. INTRODUCTION

Numerous observations of rainfall from clouds which did not reach freezing temperatures have been published in recent years (Davies 1950, Jones 1951, Kotsch 1947, Mondain 1951, Smith 1951, Styles and Cambell 1953, Virgo 1950). These were prompted by the dominance of the Bergeron theory, an idea which had in the years prior to the war received such wide acceptance that other suggested mechanisms were not considered by many to be important in the production of large amounts of rainfall.

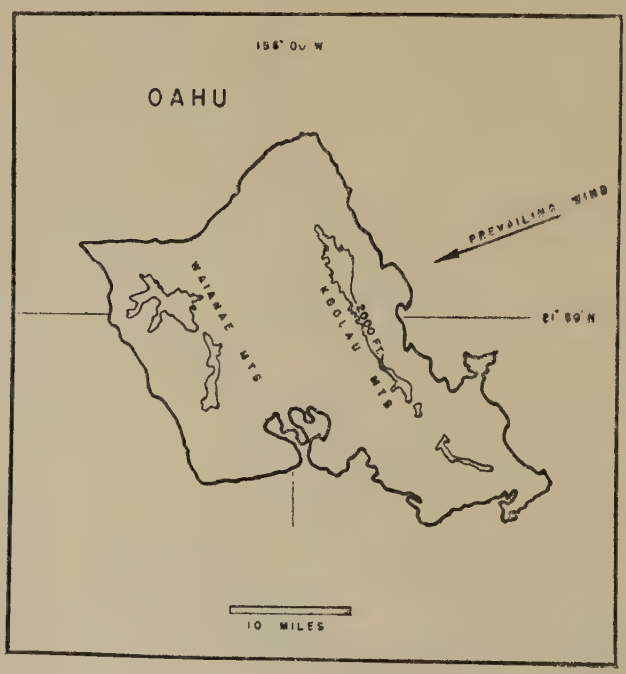


Figure 1. Island of Oahu, Hawaii.

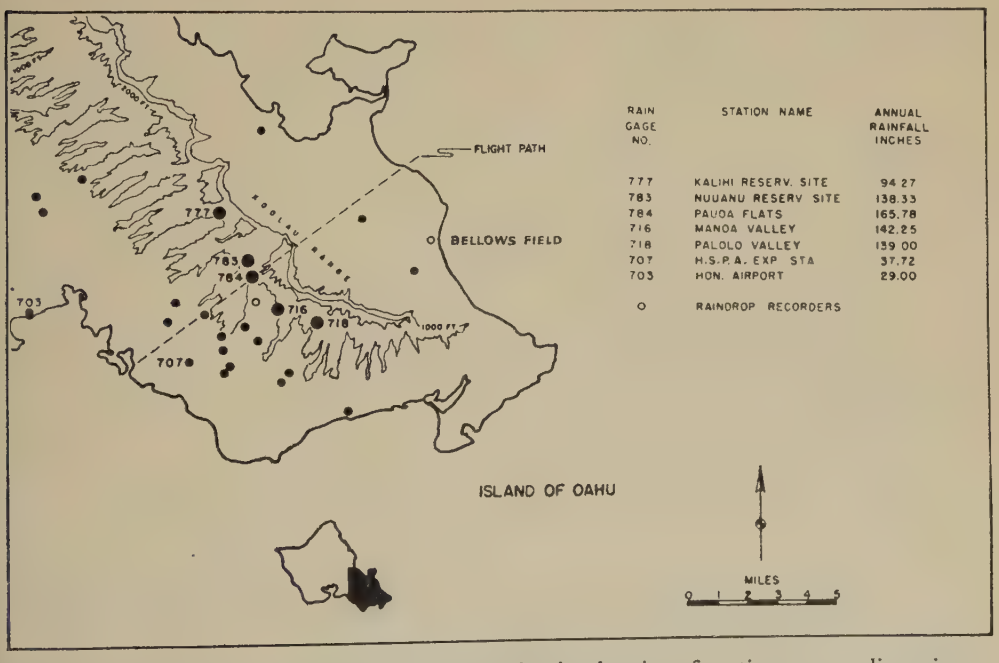
In spite of the number of accounts of warm rainfall, few details about the character of the rainfall, such as intensity, duration, drop size, and frequency, as well as cloud dimensions, have generally been described. For this reason it is felt that data presented here will be of interest to meteorologists not acquainted with rainfall conditions such as exist in Hawaii.

In the summer of 1952, a period of 10 days was selected by the staff of the Meteorology Department of the Pineapple Research Institute and the Experiment Station, Hawaiian Sugar Planters' Association, for an observational study of rainfall on the Island of Oahu, in the Hawaiian Archipelago, at 20°N, 158°W (Fig. 1). These days were 16-18 June and 25 June-1 July 1952, inclusive.

2. Description of observations

(a) *Location*

The Koolau Mountains on the Island of Oahu present a barrier almost perpendicular to the normally persistent ENE trade winds. This barrier is roughly 25 m long with a mean height of approximately 2,500 ft above msl. Some portions of the mountains extend above 3,000 ft. When the trade-winds are present, these mountains are rarely free of clouds. During trade-wind weather showers are an almost daily occurrence. On many of these days considerable rainfall occurs. Tabulation of rainfall at stations in areas receiving 140 in. annual rainfall shows that there is a yearly average of more than 250 rain-days (0·01 inch of rain or more). If all rainfall stations along the Koolau Range are considered, the number of rain-free days is about 50 per year. Typical of drier areas, a tabulation for the U.S. Weather Bureau station on Honolulu, with an average rainfall of only 30 in. per year, shows an average of more than 100 days a year with measurable rainfall.



RAIN GAGE NO.	STATION NAME	ANNUAL RAINFALL INCHES
777	KALIHI RESERV. SITE	94.27
783	NUUANU RESERV SITE	138.33
784	PAUOA FLATS	165.78
716	MANOA VALLEY	142.25
718	PALOLO VALLEY	139.00
707	H.S.P.A. EXP STA	37.72
703	HON. AIRPORT	29.00
○	RAINDROP RECORDERS	

Figure 2. South-east portion of the Island of Oahu showing location of continuous recording raingauges (double circles), standard 8 in. raingauges read daily (blacked-in circles), and Blanchard dye-treated paper tape raindrop-size recorders (open circles).

The Island of Oahu affords an excellent opportunity for this study. An area of the south-east corner of the island shown in Fig. 2 includes a large number of raingauges and meteorological stations, and is much more accessible than the other parts of the island. For these reasons observations were confined to this area.

The objective of the study was to obtain information about the meteorological conditions associated with the formation of rain from warm clouds. The observations presented here were obtained largely during aeroplane flights and at the temporary ground weather station set up at Bellows Field on the coast facing the prevailing NE winds (Fig. 2).

(b) *Cloud dimensions*

The relation of cloud depth to rainfall is a matter of increasing interest in meteorology. The heights of tops and bases of both cumulus and stratiform clouds were measured twice daily by one of the authors acting as a flight observer, at approximately 0800 and 1700 local time (LMT). Daily flights were made between Honolulu Airport and Bellows Field over the top of the Koolau Range (dashed line in Fig. 2). Results of these measurements are shown schematically in Fig. 3b. On no occasion during the ten days of observations did the measured values of the heights of cloud tops exceed 8,500 ft above msl. In general, it can be seen that the rain occurred when the cloud thickness was 5,000 ft or more. An exception to this occurred on the afternoon of 27 June when appreciable rain fell from clouds less than 4,000 ft thick.

(c) *Temperature and humidity measurements*

For clouds to qualify as warm clouds, the cloud temperature must be above 0°C everywhere and no ice crystals or frozen water drops can be present. Temperatures in the cloud layer were recorded by two aerometeorographs mounted on the aeroplane. In addition, temperatures were read from a mercury thermometer held outside the cabin of the aeroplane. Since the airspeed of the plane was only 80 mi/hr, the correction due to aerodynamic heating is only about 0·5°C. These measurements were compared with and found to agree quite well with the regular Honolulu soundings taken at 05 and 17 LMT. The Honolulu soundings are shown in Fig. 3a.

The lowest cloud-top temperature observed was 7°C. The freezing level, as determined from the soundings, was at no time below 14,000 ft. The base of the temperature inversion corresponded roughly with the tops of the clouds as observed from the aircraft. However, as much as 500 ft difference was often observed between the heights of the inversion base and of the tops of the clouds over the mountains. The tops of the more vigorous cumuli usually pushed above the inversion base, and soon evaporated in the dry air found there.

The distribution of water vapour with height was typical, throughout the period, for this region during trade-wind weather. High relative humidities (above 70 per cent) were found in the layers below the inversion base, with very dry air above. The humidity measurements were not considered accurate enough for quantitative analysis of day-to-day variation in moisture. However, the Honolulu soundings do reveal qualitatively the relation between moisture in the cloud and sub-cloud layers, and heights of cloud bases. Some comment on this will be made in a later section. One interesting fact was noted in the humidity records of the meteorographs made during level flight in the sub-cloud layer. On a number of days the humidity trace showed fluctuations of up to

30 per cent, over short distances, even though almost no variability was found in the temperatures at the same altitudes. The humidity measuring elements in these meteorographs are hair hygrometers which have considerable lag. Thus the actual spatial variation in humidity must have been even greater than that indicated.

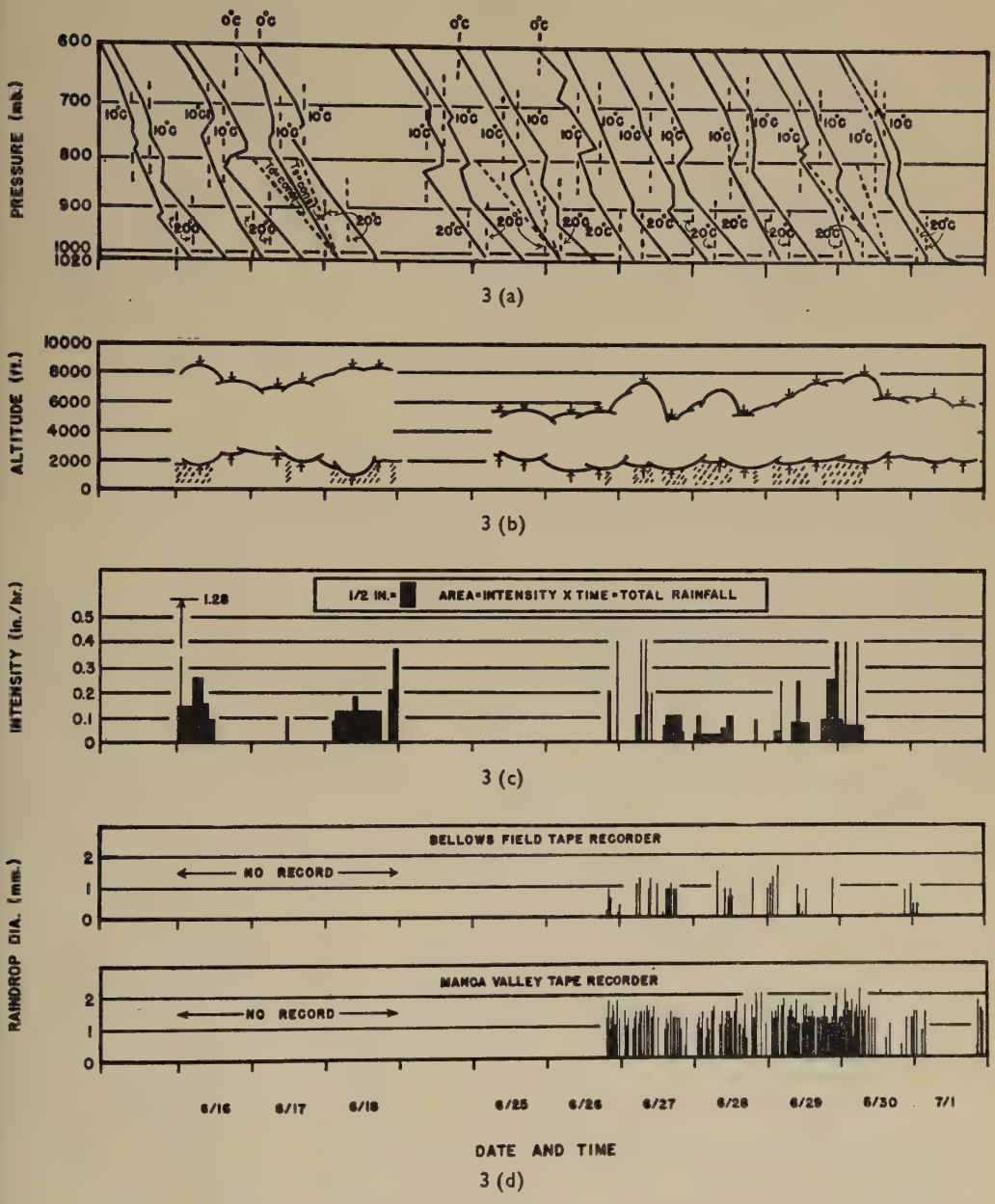


Figure 3. (a) Temperature soundings plotted for the Honolulu radiosonde station during the period of observations. Slanting dashed lines are dry (T_d) and moist (T_s) adiabats. (b) Diagram of cloud base and top measurements over the windward coast of Oahu. The short vertical arrows indicate times of measurements. The slanted dashed lines represent rainfall at one or more of the five recording raingauges. (c) Maximum rainfall intensity and duration at any one of the five recording raingauge stations (gauges located as shown in Fig. 2). (d) Time of occurrence of showers, and maximum raindrop size measured during each shower, as indicated by the Blanchard dye-treated paper tape raindrop-recorders located at Bellows Field and Manoa Valley.

(d) *Winds*

The description and analysis of the wind field is of course important in understanding details of rainfall formation. Unfortunately, the existing network of upper-wind stations in Hawaii is not adequate for analysis of the wind into divergence and vorticity components.

To supplement the regular Weather Bureau wind observations taken at Honolulu, Lihue and Hilo, five pilot balloons were released daily between 06 and 18 LMT from the Bellows Field Station. It was hoped these latter ascents would indicate variations in direction and speed of the air approaching the Koolau Range. Owing to the persistence of the orographic cloud, however, in most cases the balloon could not be followed above 2,000 or 3,000 ft.

The combined wind data show that the trade current extended at least to 10,000 ft above msl on all days of the observation period except 16 June and 30 June, when the upper limit of the easterlies was at about 5,000 ft. The variation in the wind seemed sporadic for the most part, but on 16, 17 and 18 June there was evidence of a weak easterly wave which was accompained by fine weather on 17 June and rain on 18 June. Indications of this wave can be seen in the time section of the winds for Honolulu (Fig. 4), and also in the observations at Lihue on the Island of Kauai. Normally, easterly waves passing over Hawaii are weak and difficult to detect.

It was noticed that the general visibility in the lower atmosphere varied quite closely with surface-wind speed. Measurements of atmospheric salt content by Woodcock (1953) have shown that the number of large atmospheric salt particles and also the total atmospheric salt content are much greater on days of greater wind velocity.

(e) *Rainfall observations*

The rainfall during the ten days, being the primary subject of our investigation, was analysed in considerable detail. The network of gauges used to measure daily rainfall is shown in Fig. 2 by the blacked-in circles. Most of the gauges are read at 08 LMT. The distribution of rainfall during the 10 days is shown in Fig. 5. Isohyets are drawn for 24-hour rainfall in inches. It can be seen that rain occurred each day and that more than 0·5 in. fell on six of the ten days. The largest 24-hour accumulation of rainfall occurred at Station 784 where 2·10 in. fell on 18 June.

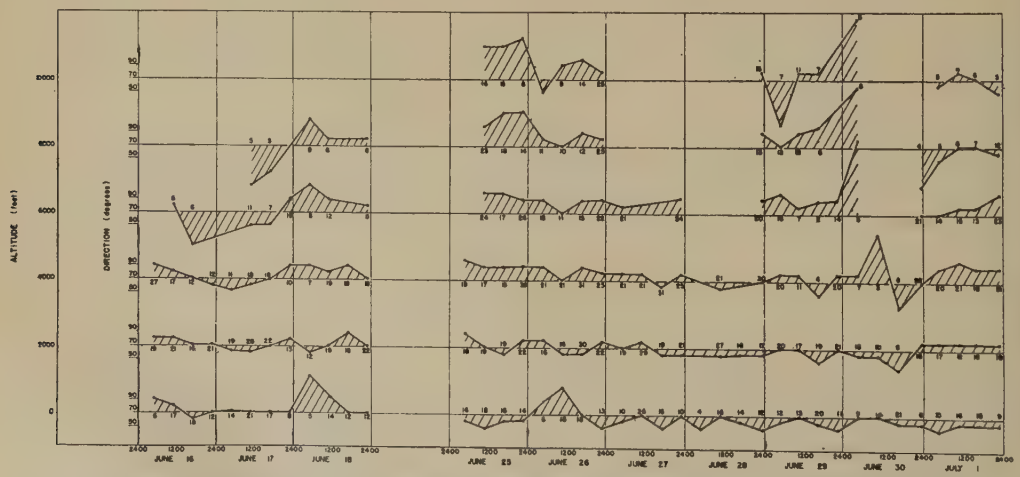


Figure 4. Time cross-section of wind speed and direction at Honolulu. Broken lines show deviation of wind direction from 70° (normal trade-wind direction) for several altitudes. Numbers next to direction graphs indicate wind speeds in knots.

Figure 5. Daily rainfall distribution for south-east Oahu. Isohyets represent rainfall amounts in inches for the 24-hour period ending 08 LMT for a given date.

Data from five Leupold-Stevens recording raingauges which were located along the lee side of the Koolau Range, as shown by the double circles in Fig. 2, were combined and analysed for a determination of rainfall intensity. The results are presented in Fig. 3c.

The intensities given are average values for the period of time indicated by the width of the bars. At 01 LMT on 16 June the rainfall intensity averaged 1·28 in./hr for 15 min and on 18, 26, 27, 29, 30 June average intensities of approximately 0·4 in./hr were observed.

Raindrop sizes were recorded during the second period of observation at two locations indicated in Fig. 2 by the open circles. The measurements were made by D. C. Blanchard (1953) with an apparatus in which a dye-treated paper tape moved beneath an upward facing 1 in. square window. The tape moved at an average rate of 14 mm/min. In Fig. 3d is shown the frequency of occurrence of showers, and the maximum size of raindrops falling during each shower, as indicated by the recorders. The drop diameters exceeded 2 mm on several occasions and 1 mm drops were common. The vertical lines in Fig. 3d represent all showers that occurred, even those which did not give measurable amounts of water in the rain gauges. This accounts for the greater frequency of rain in this figure than is indicated in Fig. 3c, in which only measurable amounts (> ·02 in.) of rain were entered. The diurnal variation in rain frequency indicated by the five recording gauges is shown in Fig. 6. The frequency of showers was greatest between 0200 and 1200 LMT. This diurnal variation has also been observed at these stations during other periods of trade-wind weather.

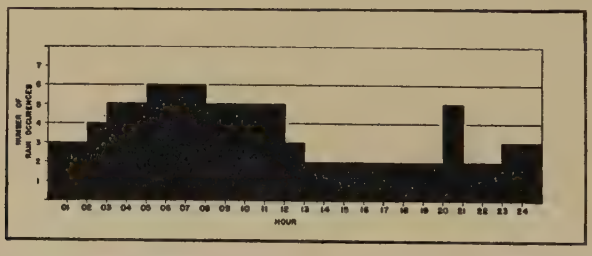


Figure 6. Diurnal variation in the frequency of rain during the 10-day period at the five recording raingauge stations along Koolau Range.

3. Further comments and discussion

Although study of the data from the ten days' observations cannot, of course, approach a comprehensive picture of the relation between warm clouds and rainfall, some features were brought out which invite consideration.

(a) *Growth of drops in clouds crossing a mountain range*

The upward surge of an air stream ascending the mountain barrier is manifest in rapidly accelerated development of the incoming clouds. The liquid-water content of these clouds also increases from condensation in this rising air. Cumuli crossing the mountain range thus experience a nearly simultaneous increase in two factors, vertical motion and liquid water, which are important for the production of rain by the coalescence process described by Bowen (1950) and others : Das 1950, Gunn and Hitschfeld 1951, Houghton 1951, Ludlam 1951, Mason 1952. The increase in the number of showers along the mountain ridge compared with that over the windward coast might be accounted for in this way.

From this reasoning, the maximum size of raindrops falling along the ridge should be greater than observed at the coast. The data from the raindrop tape recorders indicate that this is the case. The Manoa Valley instrument, located just on the lee side of the ridge, shows considerably greater rainfall frequency and larger drops than that at Bellows Field, which was almost directly upwind along the coast (Fig. 3d). The maximum drop size measured at Bellows Field for most showers ranged from 0·5 to 1·5 mm in diameter, while at Manoa the maximum drop size ranged from 1 to 2 mm in diameter.

With a normal wind speed of 15 to 20 mi/hr at the cloud-base level, the air travels from the coast to the ridge in 15 to 20 min. This time is, of course, insufficient for condensation processes to account for drop growth from 1·5 to 2·0 mm diameter. Computations of growth rate by coalescence are the right order of magnitude to account for the observed drop size. For example, using Bowen's nomograms (Bowen 1950), in 10 min a drop would grow from 0·5 mm to 1·5 mm while falling 6,000 ft through a 20 μ droplet cloud with a liquid-water content of 1 g/m^3.

(b) *Cloud types and rainfall*

The structure of the orographic cloud may be a factor in the probability of rain. In general, it may present a smooth, rounded, stratiform appearance (Figs. 7b and 7c) or it may resemble a line of cumulus (Fig. 8). In the case where the stratiform structure is seen, the cloud sometimes extends up and down wind from the ridge, forming a stratus shelf. During periods in our observational programme when this shelf was absent, or when the orographic cloud was of the cumulus type, no significant amounts of rain were recorded at the gauges along the ridge. This observation has been made casually in many instances since the observational programme reported here.

On the other hand, stratus or stratocumulus cloud layers, just below the base of the temperature inversion, occurred with dependable regularity in the 10-day period when appreciable rainfall was observed. In Fig. 7a is a cross-section of characteristic clouds over the mountain range during periods of rainy trade-wind weather. The thickness of the stratus layer was seldom more than a few hundred feet; however, misty rain or virgae were often seen falling from it, particularly at the leeward edge. Mixing of these relatively large drops with the tops of the cumuli over the mountains may be a natural 'seeding' process for the production of rain.

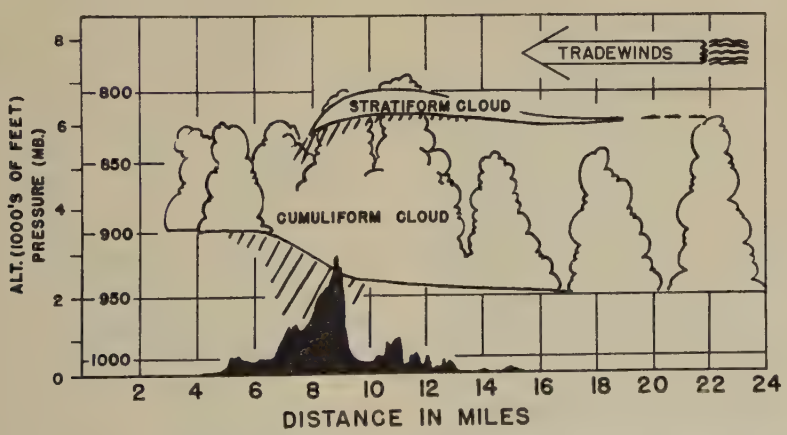


Figure 7a. Diagram of a typical cloud cross-section on a rainy trade-wind day. Stratus deck at the base of the inversion is a typical feature of such days.

While this stratiform cloud seems to be one of the contributing factors in rainfall, its appearance was not in each case associated with rain. For example, photographs of the orographic cloud taken on the afternoon flights on 26, 27 and 28 June all show the smooth stratiform top. On 27 June, however, the raingauges at stations 716 and 784 both received 0·30 in. or more precipitation between 1300 and 1900 LMT, while in the same period on 26 and 28 June no measurable rain was recorded. The incoming trade clouds during the afternoon of 26 and 28 June were small and produced no rain, while on 27 June the cumuli were well developed with numerous showers.

Observations indicate that the incoming trade clouds do not need to be raining while still at sea in order to produce significant rain when crossing the mountain ridge, but it appears that some minimum development is necessary. Certainly one of the most important factors in the formation of the heavy showers on the mountain was the mingling of the drifting trade cumuli and showers with the orographic clouds.

The amount and type of clouds appearing leeward of the orographic cloud seem to vary to a large extent with the flow pattern of the wind as it crosses the mountain range. The presence of lee waves is sometimes seen as an undulating appearance of the cloud tops. An example of this is shown by an aerial photograph (Fig. 9) taken on the afternoon of 27 June from an altitude of 7,000 ft.

(c) *Humidity*

It has been mentioned above that stratus or stratocumulus sheets at the inversion base are characteristic of rainy periods. The data also indicate that high moisture content in the lowest levels is a necessary factor for significant rainfall on the mountains. It can be noted from casual observation that rain seldom falls when the base of the orographic cloud is above the crest of the Koolau Range, that is, above 2,500 ft. This was true in all instances during the ten days of observation (Fig. 3b).

It is apparent from the foregoing that the critical conditions for orographic rainfall during a trade-wind regime are incompletely understood. A more detailed knowledge of the wind structure, both in its undisturbed state and during its passage over the mountains, is necessary to express satisfactorily its relationship to the resulting orographic cloud. This might then be combined with measurements of the physical characteristics of the incoming trade cumuli, the stratus layers and the moisture distribution in the air below the inversion, and ultimately be related quantitatively to the variation in orographic rainfall.

4. Conclusions

The occurrence of significant amounts of rainfall from warm clouds has been sufficiently well documented. The evidence here is presented as typical of the common rainfall conditions in Hawaii. The conclusions drawn are summarized as follows :

1. Large amounts of rainfall can occur from warm clouds with only moderate vertical development. Intensities as high as 0·4 in./hr were observed frequently.
2. Raindrops of more than 2 mm diameter can regularly form in clouds less than 7,000 ft thick.
3. Rainfall from warm clouds is very frequent and contributes substantially to the total rainfall for some areas.

To emphasize the importance of the contribution of warm rainfall to the total rainfall in the Islands, Table 1 shows the percentage of rainy days in each month at Station 783 (Fig. 2), which has an annual rainfall of 140 in., averaged over 46 years.

Figure 7b. Stationary stratus over Koolau Range, looking from the north-east (windward) side, with trade-wind cumulus clouds in foreground.

Figure 7c. Stratus clouds over Koolau Mountain Range, looking northward along the ridge, showing descending leeward edge.

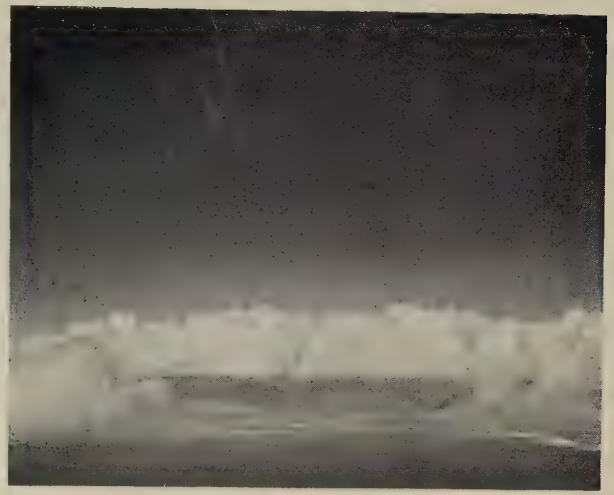


Figure 8. Stationary orographic clouds over Koolau Mountain Range, looking toward mountains from the north-east (windward) side, showing drifting trade cumuli in right and left foreground.

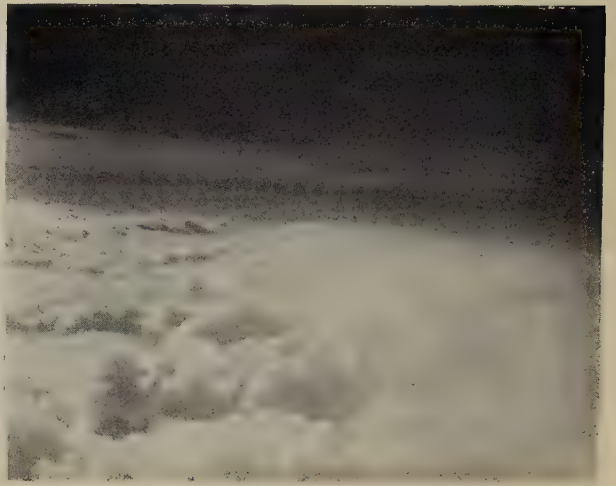


Figure 9. Leeward edge of the orographic cloud over Koolau Mountain Range, showing wave-like form of cloud tops leeward from the mountain crest. Wind is from right to left in the photograph.

TABLE 1. PERCENTAGE OF RAINY DAYS AT NUUANU (STATION 783)

Month	J	F	M	A	M	J	J	A	S	O	N	D
Percentage of days with 0·10 in. rain or more	44	42	47	53	48	56	57	60	53	50	51	51

It can be seen that 0·10 in. or more rain falls on more than half the days of the year. The number of rainy days is greater in summer than in winter.

In Table 2, the number of days when the trade-wind inversion is either absent or higher than the 750-mb level is tabulated. The figures represent the period from 1 March 1950 to 31 August 1951. At no time during the period did the freezing level over Hawaii fall below the 750-mb level and only very rarely did it occur as low as the 700-mb level.

TABLE 2. PERCENTAGE OF DAYS WITH HIGH INVERSION OVER OAHU

Month	J	F	M	A	M	J	J	A	S	O	N	D
Percentage of days with inversion above 750-mb level or absent	26	30	31	36	20	14	23	27	28	21	30	17

It can be seen from the tables that the number of days during the year when the clouds could build up to freezing levels, as indicated by the inversion data, is less than half the number of days on which rain occurs. It is therefore apparent that an important part of the total rain in Hawaii cannot be accounted for by the Bergeron process.

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Spectral line shape and its effect on atmospheric transmissions

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Summary

The detailed shape of a pressure-broadened spectral line obtained by Lindholm has been evaluated and used to derive an expression for the mean transmission of a disordered band. This is compared with the corresponding expression using the Lorentz line shape. It is shown that the difference in mean transmission for these two line shapes is scarcely detectable spectrophotometrically in the cases of the principal telluric bands of ozone and water vapour, and probably also for carbon dioxide. It is concluded that, until more important problems have been solved, it is probably uneconomical to make use of line shapes any more complicated than that of Lorentz for computations of heating rates in the atmosphere, provided always that theory is used only in order to apply existing laboratory data to atmospheric conditions.

1. Introduction

The shape of a spectral line is controlled by self-damping, Doppler effects caused by molecular motions, and collisions with other molecules. In the earth's atmosphere, self-damping is unimportant below 60 km. Doppler effects are well understood, and can be taken into account if computational facilities are available; we shall not consider them further in this paper. There remains the effect of collisions, which are essentially non-resonant collisions of absorbing molecules (principally water vapour, carbon dioxide, ozone, and, of lesser importance, rare gases such as methane and nitrous oxide) with oxygen and nitrogen molecules.

The calculation of line shapes for reasonably realistic approximations to the energy interaction in collisions has only recently been put into tractable form, and as a result most theoretical treatments of band absorption have made use of the Lorentz, or dispersion, line shape, which has the advantages that is is relatively easy to handle and that it is certainly correct near the line centre at low pressures, if allowance is made for a general shift proportional to pressure. Our object is to investigate how far the use of this line shape will lead to inaccuracies when computing atmospheric transmissions.

It must be remembered that the criterion of a ' sufficient ' representation of a line shape is that it enables the results of extensive experiments to be reduced to a few parameters (line width, line spacing, line intensity) about which theory can tell us sufficient to enable transmissions to be calculated under atmospheric conditions. According to this criterion, it is irrelevant which of the various theoretical line shapes is used in any spectral region where the absorption is due to the wings of lines outside the region, since they all happen to lead to the result that the absorption obeys Lambert's law with an absorption coefficient proportional to total pressure. Differences will arise only when the absorption coefficient and its temperature dependence are calculated theoretically, but in all cases this would be a very suspect calculation, and a few experimental measurements used with the qualitative theoretical result would be greatly preferred.

In a region where the absorption lines are strong it is usual to calculate the mean absorption as if the region formed part of an infinite homogeneous array of absorption lines. In this case, which is the one that will be treated here, different line shapes lead to qualitatively different transmission functions. If the problem arises of weak lines with a background due to the wings of distant lines, it should be possible to analyse the experimental results in terms of the product of a Lambert's law term and a transmission function as derived here.

An attempt to treat the problem of the effect on band transmission of deviations from the Lorentz shape was made by Plass and Warner (1952 a and b), but this is unacceptable for the following reasons :

(i) Two models of absorption bands are used. The first, a single-line model, is misleading when discussing this problem since it differs from any realistic model in the very much greater emphasis which it places on the exact shape of the far wings of a line. An Elsasser model for regularly spaced lines is also discussed (1952 a) but here the authors assume that the asymptotic expressions for the shape of the line wings are valid right to the line centre, where they diverge.

(ii) Of less critical importance, the shape which they use for discussion of the single-line model is not in accordance with the most complete of modern theories (Lindholm 1945), and, in particular, it is not normalized, so that the oscillator strength is pressure-dependent. Errors of this nature lead to difficulties which are discussed below.

In this paper we shall treat only the problem of constant-pressure paths, since the difficulties are greatly increased by attempting to treat the atmospheric problem in its entirety. This incidentally permits us to ignore the pressure shift of the line centres provided that the shift is approximately the same for all lines, so that the mean line spacing is not pressure-dependent. We shall therefore not be able to reach any conclusion about the effect of pressure shift on atmospheric computations.

2. Preliminary investigation

For a preliminary investigation we adopted a rough line shape similar to that used by Plass and Warner. This is conveniently described in terms of a frequency ν_p, derived from the Lindholm theory, which is independent of pressure and is given by

$$\nu_p = \tfrac{3}{64}\,(8/3\pi^2)^{1/5}\, b^{-1/5}\, v^{6/5}$$
$$= 0{\cdot}0361\, b^{-1/5}\, v^{6/5} \qquad . \quad . \quad . \quad . \quad (1)$$

Here v is the mean relative molecular velocity defined by

$$v = \left[\frac{8kT}{\pi}\left(\frac{1}{m} + \frac{1}{m_0}\right)\right]^{1/2} \qquad . \quad . \quad . \quad (2)$$

where k is Boltzmann's constant, T is the absolute temperature, m, m_0 are the masses of the perturbing and absorbing molecules, and b is the van der Waals constant defined by the expression for the instantaneous frequency perturbation, $\Delta\nu = b/R^6$, R being the intermolecular distance. Here and throughout this paper, frequency means number of vibrations/sec, not of radians/sec; except that where specifically stated (e.g., in Table 1) it is given in wave numbers. The question of the validity of an inverse sixth-power perturbation in atmospheric applications will be discussed later.

For the evaluation of ν_p one is faced with the problem of the exact value of the van der Waals constant, which is unknown, However, the Lindholm theory also gives for the half-line width

$$\alpha = 1{\cdot}34\, b^{2/5}\, v^{3/5}\, N \qquad . \quad . \quad . \quad . \quad (3)$$

where N is the number of perturbing molecules/cm^3 (so that α is proportional to pressure). Since we have some spectroscopic measurements of α for atmospheric bands, it is obviously appropriate to calculate ν_p from α by eliminating b. There results

$$\nu_p = 0{\cdot}0418\, (v^3\, N/\alpha)^{1/2} \qquad . \quad . \quad . \quad . \quad (4)$$

Estimated values of ν_p, together with other important parameters, are given for various infra-red molecular bands in Table 1.

TABLE 1. ESTIMATES OF THE VALUES OF PARAMETERS

Gas	Band	Model*	δ†	α‡	ν_p	$y = \frac{\alpha}{\delta}$	$z = \frac{\delta}{\nu_p}$	$\epsilon = yz$
			cm^{-1}	(cm^{-1} at STP)	(cm^{-1} at 273°K)	(at STP)		(at STP)
H_2O	6·3 μ rotation	S.M.	2·0	0·11	2·4	0·055	0·82	0·045
CO_2	15 μ	E.M.	1·56	0·12	1·7	0·077	0·92	0·071
	4·3 μ							
N_2O	17 μ	E.M.	0·84	0·14	1·5	0·17	0·56	0·093
	7·8 μ							
O_3	9·6 μ	S.M.	0·09	0·1	1·8	1·1	0·050	0·056
CH_4	3·3 μ	E.M.	9·2	0·09	2·8	0·010	3·3	0·032
	7·6 μ	S.M.	1·0	0·09	2·8	0·09	0·36	0·032

* S.M. = statistical model for randomly-spaced lines. E.M. = Elsasser model for regularly-spaced lines. The statistical model has been justified for water-vapour bands by Goody (1952) and seems to be a reasonable approximation for the 9·6 μ ozone band (Walshaw and Goody 1953). The Elsasser model is the obvious choice for the cases where it is used (see, for example, Herzberg 1945). The statistical model would appear to be the more likely for the 7·6 μ methane band.

† The value of δ, the mean line spacing, for water vapour varies over the band; 2·0 cm^{-1} is the value which gives the best fit to the 'universal' curve proposed by Cowling (see Goody 1952). For the Elsasser models δ is obtained by inspection of an absorption spectrum. Walshaw and Goody (1954) have measured y for the centre of the 9·6 μ ozone band, and δ is chosen to be consistent with this and the assumed value of α. For the 7·6 μ methane band the value of δ has been guessed.

‡ The line widths suggested are intended to be reasonable mean values for a quantity which can vary somewhat with the rotational quantum number; they all refer to a condition of infinite dilution in air. For water vapour we have the direct observations of Adel (1947) and Becker and Autler (1946) on the rotation band. Adel (1953) has recently measured a line width in the 15 μ band of carbon dioxide; he suggests that this may be a slight overestimate. Goody and Wormell (1951) found $\alpha = 0{\cdot}15$ cm^{-1} for the 7·8 μ band of nitrous oxide, and also gave data from which the line width for infinite dilution in air can be deduced from the figures for self-broadening given by Adel and Barker (1944) and Thorndike (1948). When adjusted to these conditions the work of Adel and Barker on the 17 μ band gives $\alpha = 0{\cdot}16$ cm^{-1}, which may be an overestimate according to Adel (1947). Thorndike's work gives $\alpha = 0{\cdot}17$ cm^{-1} for the 17 μ band, which may be in error, and $\alpha = 0{\cdot}10$ cm^{-1} for the 7·8 μ band. The best average for our purpose has been taken to be 0·14 cm^{-1}. The width of a line in the 1·7 μ methane band has been measured by Goldberg (1951). No line widths have been measured in the ozone spectrum, but bearing in mind the approximate proportionality which exists between optical and kinetic theory collision diameters for non-resonant collisions (Coggeshall and Saier 1947), it seems likely that the figure given in the table is correct within a factor of two.

The rough line shape used consisted of a Lorentz centre out to frequencies $\pm a\nu_p$ from the line centre, beyond which the red and violet wing approximations used by Plass and Warner were employed. The constant a was chosen to make the intensity of the line independent of pressure, and it is satisfactory that this gives a value of 0·587, since the work of Spitzer (1940) and Holstein (1950) indicates a change of regime at a frequency of order ν_p.

It should be pointed out at this stage that pressure independence of the line intensity, i.e., the oscillator strength, is a strict necessity for all treatments which have been made of the problem of pressure broadening. They nearly all confine themselves to the classical theory of adiabatic collisions, but even when splitting of energy levels is taken into account it is assumed that the total multiplet strength is invariant (e.g., Spitzer 1940). This does not, of course, mean that it is impossible for oscillator strengths to vary under pressure, but it is unlikely at moderate pressures. In fact, measurements of vibrational oscillator strengths made at high pressures appear to agree well with the values obtained from infra-red dispersion measurements (see for example Penner and Weber 1951, who worked on the fundamental carbon monoxide band at pressures up to 50 atm). Lastly the question

of ' quenching ' (see for example Margenau and Watson 1936) is irrelevant in atmospheric problems below 60 km, which with respect to translational, vibrational and rotational energy levels involve thermodynamic equilibrium, while ' quenching ' involves a lack of detailed balance between excitation and de-excitation processes.

In order to treat the problem of band absorption, we will make use of the statistical model of a band, which assumes random line positions, with intensities distributed according to an exponential law (Goody 1952). It is possible to substitute arbitrary line shapes which are independent of line position into Eq. (3) of that paper, provided that the integrations are carried out separately for the red and blue wings. We did this, and found to a high degree of approximation

$$z \log_e (T_L/T) = 1{\cdot}268\, \theta'^2/(1 + 2{\cdot}947\, \theta'), \quad \theta' \leqslant 3,$$
$$\sim 2{\cdot}307\, \theta'^{2/3} + 1{\cdot}138\, \theta'^{3/7} - 3{\cdot}142\, \theta'^{1/2}, \quad \theta' \text{ large} \quad . \quad . \quad (6)$$

where T is the mean transmission, T_L is the mean transmission which would have been obtained had the lines had the Lorentz shape throughout, $z = \delta/\nu_p$, $\theta' = \sigma m\alpha/\pi\nu_p^2$, δ being the mean line spacing, σ the mean line intensity, and m the amount of absorbing matter. From this computation we were able to deduce the following results :

(i) The difference $(T_L - T)$ decreases to zero with increasing pressure. In fact, for any model of a band the transmission at sufficiently high pressures depends only on the ratio of mean line intensity to mean line spacing. If the oscillator strength varied with pressure, large differences could occur at high pressures; but as we have seen above, this does not appear to happen in practice.

(ii) As the pressure tends to zero the difference tends to a limit, which can be maximized with respect to m, to give an upper bound for the difference as a function of z. This upper bound is shown in Fig. 2 under the title ' Preliminary line shape.'

(iii) There is a rapid increase of the maximum difference at about $z = 0{\cdot}3$. This led us to the conclusion that the exact line shape for frequencies greater than about δ from the line centre was not important compared with that within these limits, presumably because of overlapping by other lines. The shape of the transmission curve for $z < 1$ might therefore be expected to depend critically on the precise way in which the line centre approximation changes over to the line wing approximations. Since the range $z < 1$ is the important one for atmospheric bands we decided that a more realistic shape should be investigated.

3. The shape of a pressure-broadened line

The only treatment of the shape of a pressure-broadened line which is not restricted to any particular range of pressures or distances from the line centre is that of Lindholm (1945). His derivation is based on the classical Fourier-integral method for an adiabatic inverse sixth-power frequency perturbation. Having no more detailed knowledge of the particular interaction forces involved, one would naturally accept an inverse sixth-power law as a first approximation for non-resonant interactions in the atmosphere despite the fact that it cannot hold for very close collisions, and the Fourier-integral approach now seems to be generally accepted by workers in this field. Lindholm introduces certain other simplifying approximations, but his derivation appears to be sound, particularly as regards frequencies whose distance from the line centre is not large compared with ν_p. For these reasons the Lindholm shape was adopted as being far more accurate than the Lorentz shape.

Lindholm gives his line shape in terms of two parameters h and k which can be written as follows :

$$h = \alpha/0{\cdot}795\,\nu_p, \qquad k = (\nu - \nu_0)/\nu_p \quad . \quad . \quad . \quad . \quad . \quad (7)$$

We shall find it more convenient to use

$$\epsilon = 0{\cdot}795\,h, \qquad \mu = k + 0{\cdot}577\,h \quad . \quad . \quad . \quad . \quad (7a)$$

Then the line shape $S(\nu - \nu_0)$ which has to be used is given by

$$\pi S(\nu - \nu_0)\,d\nu = (I_0 + I_\infty)\,dk = (I_0 + I_\infty)\,d\mu \quad . \quad . \quad (8)$$

where I_0 and I_∞ are defined by Lindholm's equations (31) and (32). Now inspection of the last column of Table 1 shows that in the atmosphere ϵ is numerically small at one atmosphere pressure, and it decreases with pressure. It is therefore possible to expand Eq. (8) and retain only the terms of first order in ϵ, with the exception that the denominator of I_∞ is kept as $(\mu^2 + \epsilon^2)$, instead of being approximated to by μ^2, to avoid introducing a spurious divergence at $\mu = 0$. There is a term in I_∞ of zero order in ϵ, but this is cancelled by one in I_0. It is to be noted that nowhere do we assume μ small, so the resulting line shape is valid for all μ, i.e. throughout the frequency range, subject only to the restriction $\epsilon \ll 1$. With this approximation we have

$$(\mu^2 + \epsilon^2)\,I_\infty = \epsilon\,[(1 - 0{\cdot}285\,\mu)\cos\mu + 1{\cdot}208\,\mu\sin\mu] - \mu\sin\mu, \quad . \quad (9)$$

$$\begin{aligned}(\mu^2 + \epsilon^2)\,I_0 &= \mu\sin\mu + 0{\cdot}726\,\epsilon\,[\sin\mu - \mu\cos\mu] - 1{\cdot}051\,\epsilon\,J(\mu)\\ &\quad - 0{\cdot}199\,\epsilon\,\mu^2\,[F(\mu)\cos\mu + G(\mu)\sin\mu]\\ &\quad - 0{\cdot}115\,\epsilon\,\mu^2\,[F(\mu)\sin\mu - G(\mu)\cos\mu], \quad . \quad . \quad . \quad (10)\end{aligned}$$

where*

$$F(\mu) = 1 - \frac{(3\,\mu)^2}{10.13} + \frac{(3\,\mu)^4}{10.13.16.19} - \ldots \quad . \quad . \quad . \quad (11)$$

$$G(\mu) = \frac{3\,\mu}{10} - \frac{(3\,\mu)^3}{10.13.16} + \ldots \quad . \quad . \quad . \quad . \quad (12)$$

$$J(\mu) = \mu\sin(\mu - \pi/4) + \tfrac{1}{2}(\pi\mu)^{1/2}\,[C(\mu) - S(\mu)] \quad . \quad . \quad (13)$$

and $C(\mu)$, $S(\mu)$ are the Fresnel integrals defined by

$$C(\mu) - iS(\mu) = \int_0^\mu (2\pi u)^{-1/2}\,e^{-iu}\,du. \quad . \quad . \quad . \quad (14)$$

Since in the present notation the pressure-shifted Lorentz line shape is given by

$$S_L(\nu - \nu_0)\,d\nu = \frac{\epsilon\,d\mu}{\pi(\mu^2 + \epsilon^2)} \quad . \quad . \quad . \quad . \quad (15)$$

Eqs. (8), (9) and (10) enable us to express the ratio S/S_L as a function of μ. This function has been computed for $-4 \leqslant \mu \leqslant +4$ and is plotted in Fig. 1 (heavy line). Also shown in Fig. 1 are the straight line

$$S/S_L = 1 - 0{\cdot}286\,\mu \quad . \quad . \quad . \quad . \quad . \quad (16)$$

taken from Lindholm's low-pressure line centre approximation (Eq. (35) of his paper), which is tangent to the exact curve at $\mu = 0$, and the Lindholm line wing approximations,

* $F(\mu)$, $G(\mu)$ are the real and imaginary parts of a function which can be expressed in terms of an incomplete Γ-function of purely imaginary argument :

$$F(\mu) + iG(\mu) = \tfrac{7}{3}\,\mu^{-7/3}\,e^{i\mu}\int_0^\mu u^{4/3}\,e^{iu}\,du$$

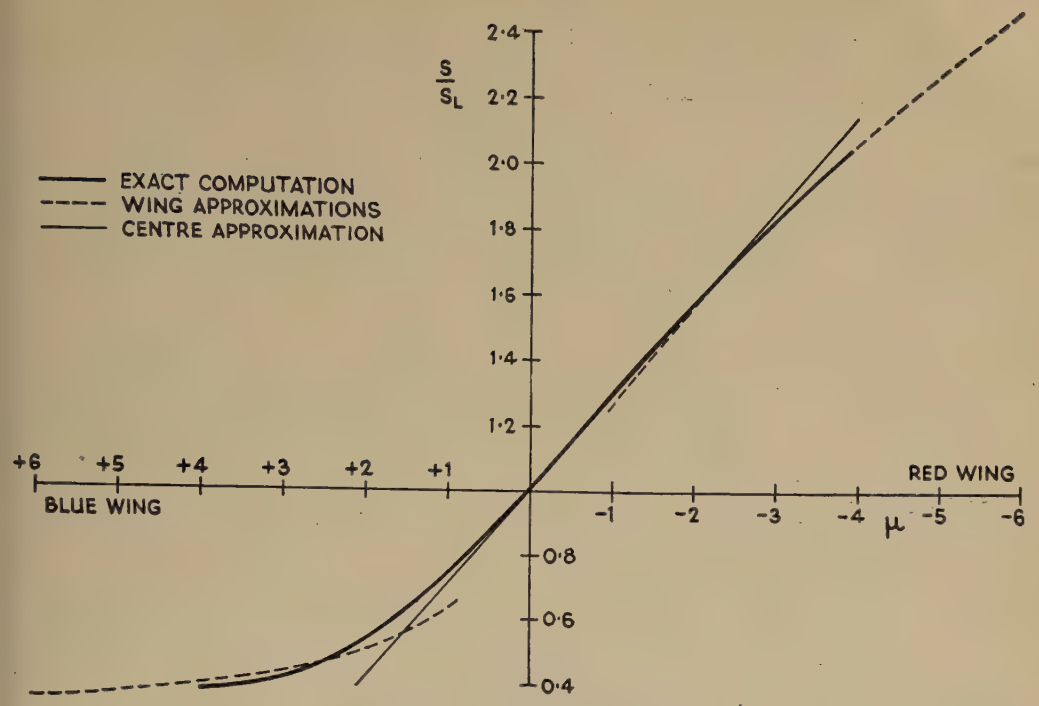

Figure 1. Lindholm's line shape.

to first order in ϵ, taken from Eqs. (38) and (39) of his paper. In the present notation these are given by

$$\left.\begin{aligned} S/S_L &= 0{\cdot}932\,|\mu|^{1/2} + 0{\cdot}319\,|\mu|^{-1/3} \quad \text{(red wing)} \\ S/S_L &= 0{\cdot}638\,\mu^{-1/3} \quad \text{(blue wing)} \end{aligned}\right\} \quad . \quad . \quad (17)$$

It is seen that the red-wing approximation fits excellently both to the exact curve and to the straight line approximation, while the blue-wing approximation fits fairly well to the exact curve. The discrepancy in the blue wing is within Lindholm's estimate of the errors in his wing approximations, which on this scale is $0{\cdot}09/\mu$.

For the purpose of evaluating the mean transmission it is desirable to use the simplest line shape that can be considered an adequate representation, and with this consideration in mind we adopted the straight line for $-2{\cdot}5 \leqslant \mu \leqslant +1{\cdot}5$ and the Lindholm wing approximations outside this range. The line so chosen does not satisfy the condition that the intensity is independent of pressure, and indeed we should not expect it to, for it can be made to do so by a change of order ϵ^2 in the line centre, and we have neglected terms of this order. The required change is to multiply the absorption coefficient in the region μ small by the factor $(1 + 0{\cdot}193\,\epsilon)^{-1} \simeq (1 - 0{\cdot}193\,\epsilon)$, and this agrees fairly well with the third term in the numerator of Lindholm's equation (35) for the line centre, which would give a factor $(1 - 0{\cdot}208\,\epsilon)$. The difference between this and the factor we have found necessary can presumably be explained by the fact that the shape we have adopted is not quite the exact shape, especially in the blue wing. As has been stated, it is sufficient for normalization (to order ϵ) if the absorption coefficient in the line centre is multiplied by this factor; but for simplicity we take the absorption coefficient to be multiplied by it everywhere, the errors thus introduced in all parts of the line being of the same order as the terms neglected. This in fact then gives exact normalization.

We finally adopt, therefore, the line shape

$$\left.\begin{aligned}
&\text{Red wing :} \quad S(\nu-\nu_0) = \frac{\epsilon}{\pi\nu_p}\,\frac{0{\cdot}932\,|\mu|^{-3/2} + 0{\cdot}319\,|\mu|^{-7/3}}{1+0{\cdot}193\,\epsilon}\,, \quad \mu < -2{\cdot}5\\
&\text{Centre :} \quad\;\; S(\nu-\nu_0) = \frac{\epsilon}{\pi\nu_p}\,\frac{1-0{\cdot}286\,\mu}{(1+0{\cdot}193\,\epsilon)\,(\mu^2+\epsilon^2)}\,, \quad -2{\cdot}5 \leqslant \mu \leqslant +1{\cdot}5\\
&\text{Blue wing :} \quad S(\nu-\nu_0) = \frac{\epsilon}{\pi\nu_p}\,\frac{0{\cdot}638\,\mu^{-7/3}}{1+0{\cdot}193\,\epsilon}\,, \quad \mu > +1{\cdot}5
\end{aligned}\right\} \quad (18)$$

4. The transmission of a statistical band with Lindholm lines

When we insert the line shape (18) into Eq. (3) of Goody's (1952) paper, we obtain

$$-z\log_e T = \int_{-2{\cdot}5}^{1{\cdot}5} \frac{\theta\,(1-0{\cdot}286\,\mu)\,d\mu}{\mu^2+\epsilon^2+\theta\,(1-0{\cdot}286\,\mu)} + \int_{2{\cdot}5}^{\infty} \frac{\theta\,[0{\cdot}932\,\mu^{-3/2}+0{\cdot}319\,\mu^{-7/3}]\,d\mu}{1+\theta\,[0{\cdot}932\,\mu^{-3/2}+0{\cdot}319\,\mu^{-7/3}]}$$
$$+ \int_{1{\cdot}5}^{\infty} \frac{0{\cdot}638\,\theta\mu^{-7/3}\,d\mu}{1+0{\cdot}638\,\theta\mu^{-7/3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where

$$\theta = \sigma m\alpha/[\pi\nu_p{}^2\,(1+0{\cdot}193\,\epsilon)] \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The first integral on the right-hand side can be evaluated exactly, giving

$$K_1(\theta) = \frac{\theta\,(1-0{\cdot}0408\,\theta)}{[\epsilon^2+\theta\,(1-0{\cdot}0204\,\theta)]^{1/2}}\left[\pi - \tan^{-1}\frac{4\,[\epsilon^2+\theta\,(1-0{\cdot}0204\,\theta)]^{1/2}}{3{\cdot}75-1{\cdot}143\,\theta-\epsilon^2}\right]$$
$$+ 0{\cdot}132\,\theta\log_e\left[\frac{6{\cdot}25+\epsilon^2+1{\cdot}715\,\theta}{2{\cdot}25+\epsilon^2+0{\cdot}571\,\theta}\right] \quad . \quad . \quad . \quad . \quad (21)$$

The second and third integrals are transformed by the substitution $\mu = 1/\eta$ and integrated once by parts. The integrated terms then give the greater contribution, and the remaining integrals can be evaluated with sufficient accuracy by Simpson's rule with three ordinates, for $\theta \leqslant 2$. The contribution from the second and third integrals is

$$K_2(\theta) = \frac{1{\cdot}248\,\theta}{1+0{\cdot}274\,\theta} + \frac{0{\cdot}0917\,\theta^2}{(1+0{\cdot}274\,\theta)^2} + \frac{0{\cdot}1637\,\theta^2}{(1+0{\cdot}0908\,\theta)^2} + \frac{0{\cdot}278\,\theta}{1+0{\cdot}247\,\theta}$$
$$+ \frac{0{\cdot}0268\,\theta}{(1+0{\cdot}247\,\theta)^2} + \frac{0{\cdot}0169\,\theta^2}{(1+0{\cdot}0492\,\theta)^2}\,, \quad 0 \leqslant \theta \leqslant 2 \quad . \quad (22)$$

The Lorentz expression which has to be subtracted to give $z\log_e(T_L/T)$ is

$$K_3(\theta) = -z\log_e T_L = \pi\theta'/(\epsilon^2+\theta')^{1/2} \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where

$$\theta' = \theta\,(1+0{\cdot}193\,\epsilon) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Then

$$z\log_e(T_L/T) = K_1(\theta) + K_2(\theta) - K_3(\theta) = K(\theta), \text{ say} \quad . \quad . \quad . \quad . \quad (25)$$

where we find easily

$$K(\theta) \sim 0{\cdot}606\,\theta^2/\epsilon^2, \qquad \theta \ll \epsilon^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

$$K(\theta) \sim K_2(\theta) - \frac{\theta^{1/2}(1 - 0{\cdot}0408\,\theta)}{(1 - 0{\cdot}0204\,\theta)^{1/2}} \tan^{-1} \frac{4\,\theta^{1/2}(1 - 0{\cdot}0204\,\theta)^{1/2}}{3{\cdot}75 - 1{\cdot}143\,\theta}$$

$$+ 0{\cdot}143\,\theta \log_e \frac{6{\cdot}25 + 1{\cdot}715\,\theta}{2{\cdot}25 + 0{\cdot}571\,\theta} - \pi\,\theta^{3/2}(0{\cdot}0306 - 0{\cdot}00026\,\theta)$$

$$- 0{\cdot}303\,\epsilon\,\theta^{1/2}, \quad \epsilon^2 \ll \theta \leqslant 2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

The expression (27), with the last term omitted, is an upper bound to $K(\theta)$ for all values of ϵ, and is its limit as $\epsilon \to 0$. In the range $0 \leqslant \theta \leqslant 2$ it is closely approximated to by

$$K(\theta) = 0{\cdot}606\,\theta/(1 + 0{\cdot}158\,\theta^{1/2}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

To the same approximation, namely $\epsilon^2 \ll \theta$, we have

$$-z \log_e T_L = \pi\theta'^{1/2} \to \pi\,\theta^{1/2}, \text{ as } \epsilon \to 0 \quad . \quad . \quad . \quad . \quad . \quad (29)$$

and hence

$$T_L - T \to \exp(-\pi\,z^{-1}\,\theta^{1/2}) \left[1 - \exp\left(-\frac{0{\cdot}606\,\theta\,z^{-1}}{1 + 0{\cdot}158\,\theta^{1/2}}\right)\right] . \quad . \quad (30)$$

as $\epsilon \to 0$. This expression is a maximum at approximately the value θ_m of θ which satisfies

$$z = \frac{\pi}{2}\theta_m{}^{1/2}(1 + 0{\cdot}158\,\theta_m{}^{1/2})/(1 + 0{\cdot}079\,\theta_m{}^{1/2}) \quad . \quad . \quad . \quad (31)$$

and it is seen that θ_m is a monotonic increasing function of z, which remains within the range $0 \leqslant \theta \leqslant 2$ for $0 \leqslant z \leqslant 3{\cdot}54$, a range which is ample for our purpose. We used θ_m as a parameter, from which we determined values of z by (31), and thence values of $(T_L - T)_{\max}$ for these values of z from (30), and thus plotted the curve of Fig. 2 marked ' Lindholm's line shape.' The maximum differences for the case $\epsilon = 0{\cdot}1$ have also been calculated as a function of z, and they lie close to and below this limiting curve. It is worth noting that the maximum values of $(T_L - T)$ all occur at small transmissions varying from 0·135 to 0·165, depending on the value of z.

5. Discussion

The differences of mean transmission plotted in Fig. 2 are the maximum which would be observable in any circumstances, according to the data used. They are : 0·2 per cent for the 9·6 μ ozone band; 1·1 per cent for the 7·6 μ methane band; 2·4 per cent for water-vapour bands (see notes on Table 1). In practice, an experimentalist when fitting his results to theoretical predictions does not allow all his errors to accumulate at one point, but produces the best fit with roughly equal positive and negative errors. Further, he is likely to be mainly interested in band areas, and the averaging effect will again decrease the observable error. In all, it is unlikely that he will be called upon to detect differences much greater than about $(T_L - T)/4$. It is not, therefore, surprising that the Lorentz line shape has been found adequate to explain laboratory measurements of mean transmissions, since these are rarely made at an accuracy of much greater than one per cent, particularly for transmissions not near unity. It is conceivable that an experiment to verify Eq. (30) could be designed by working at very small transmissions, where $\log(T_L/T)$ can be made large.

The question of importance to atmospheric studies is whether errors induced by assuming a Lorentz line shape are significant when computing heating effects. For this

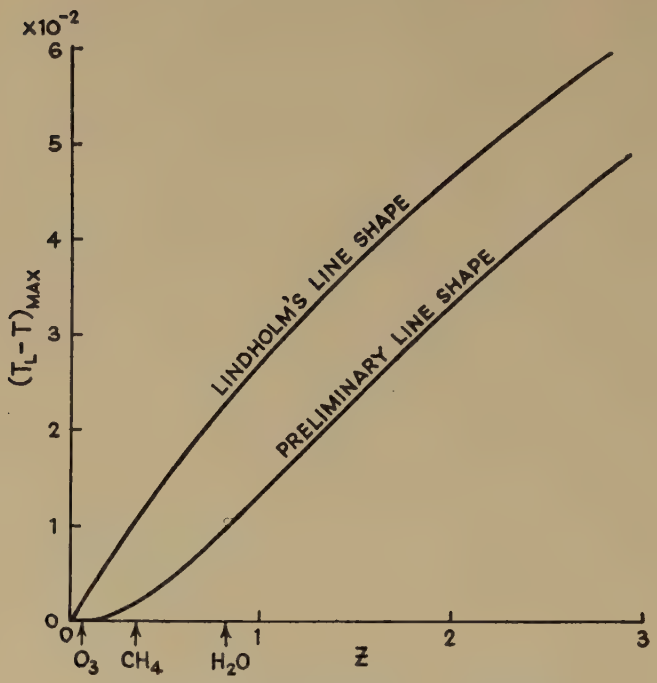

Figure 2. Maximum differences of transmission.

purpose such functions as the derivatives of the transmission with respect to path length are more significant than the transmission itself. It is however very difficult to set up any satisfactory criterion for a real atmosphere with a vertical pressure gradient. Moreover, it would be unrealistic to examine the problem for one spectral range only when there is bound to be cancellation of errors when the frequency integration required to find the total flux divergence is performed. A conclusion could be reached only by constructing radiation charts identical in all respects except for the form of the transmission function and comparing the heating rates computed from them.

This we cannot do at present, so we can only state a tentative opinion on the importance of the exact line shape. It is known that larger errors in transmission than those discussed in this paper arise from :

(*a*) use of incorrect models for absorption bands;

(*b*) failure to take proper account of the change of line width (and perhaps line position) with pressure;

(*c*) poor laboratory data.

This suggests that as a matter of practical policy it would be wise to satisfy oneself on these matters first before introducing the great complication of new line shapes. We believe moreover that after the averaging required to compute total flux divergence the effects of deviations from the Lorentz line shape on atmospheric heating could be quite small, and that it would be a pity if more important improvements to radiation charts were held up while this problem was investigated.

The above remarks apply only to those bands for which the statistical model is appropriate. There are three examples in Table 1 for which the regular Elsasser model would be far more suitable, and of these carbon dioxide is of very great importance in the atmosphere. The mathematical difficulties of the Elsasser model are much greater than those of the statistical model; but because of the regular spacing of the lines the

effect of the far wings will be less important for similar values of z, and we accordingly expect that the errors in assuming a Lorentz shape will not be greater for the Elssaser model than for the statistical model. Even if we assume them to be the same, we see that the maximum error in the important case of carbon dioxide is about 3 per cent, which is probably not serious.

The 3·3 μ methane band is of no thermal importance in the atmosphere, but if evidence is to be sought for deviations from the Lorentz line shape this would seem to be the most suitable band for experimental study, because of the large value of z.

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The nature of convection as revealed by soaring birds and dragonflies

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SUMMARY

Birds and dragonflies can be used as indicators of the extent and intensity of vertical currents, particularly of the nature of convection below cloud base during its period of growth. The bubble theory of Scorer and Ludlam is used to interpret the observations of Hankin in India. The possibility of deriving information from the flight of swallows in Britain is also discussed. In hot sunshine and no wind the air appears to stream steadily upwards from fixed places for the first few metres. The birds indicate that convection develops upwards, the heavier ones which soar at greater heights taking to the air last. The upcurrents are stronger and wider at greater height and drift with the wind; they react more slowly at greater height to changes of sunshine on the ground. When convection is intense the upcurrents appear more sharp-edged. Puffs of wind are associated with the descent of air from above the superadiabatic layer to replace a part of that layer that is ascending as a newly-created bubble. When the air is stirred mechanically by wind, bubble formation in the lowest layers is reduced.

It is suggested that air movements on the scale of ordinary convection are an element in the climate as important as any other in the life of many birds.

1. INTRODUCTION

Our ideas of how the air moves when convection is taking place are based largely upon observation of cumulus clouds and the experience of glider pilots. Laboratory experiments are analogous to special kinds of convection only, and in the layers below cloud base, evidence is scanty. Soaring birds, when they occur in sufficient numbers, may reveal the nature of the motion close to the ground, but they are not common in Britain, nor are they as large as in hot climates. Large birds do not engage in prolonged flapping flight; swans, and some wading birds engage in it very little, while ostriches cannot fly at all. These larger birds have been able to give a greater proportion of their muscular weight to strength of beak and claw and those that fly extensively engage almost entirely in soaring.

A record of a few years observation of the flight of soaring birds in India was made by Hankin (1913) who was interested in all aspects of animal flight. In spite of his careful observations and his knowledge that some informed opinion ascribed the bird's power of soaring to thermal upcurrents he was convinced that this opinion was erroneous, though he could not himself explain soaring. This difficulty was mainly due to some preconceived ideas about thermals, and a gross overestimate of the sinking speed of the gliding birds. He found the meteorological texts of the times most unhelpful, and depended on his own wits and knowledge of aeronautics to classify his observations.

The recently developed bubble theory of convection (Scorer and Ludlam 1953) tackled the difficulty existing in previous bubble theories of how bubbles originate and since no evidence on which it can be tested has been collected by aircraft other sources of information were sought. Hankin's book provides the best-known description of the habits and behaviour of soaring birds and is a valuable source even though it is now universally agreed that the birds soar in thermals.

After a brief summary of those of Hankin's observations which are relevant to the motion of the atmosphere, and a discussion of how swallows may be usefully observed in Britain, the bubble theory is applied in an attempt to give coherence to what is seen.

Hankin's observations were made mainly using binoculars. He also observed the tracks of birds by marking them on a horizontal mirror. Any meteorologist reading his book will wish that he had as much time for this pursuit. Three extracts from his diary are given in an appendix to illustrate the close connection between the behaviour of birds and dragonflies and air movements.

2. Summary of Hankin's observations*

(a) *The flight of soaring birds*

Soaring birds (Table 1 of Appendix) begin their flight by flapping up to a height which is, in the case of vultures, 10-20 m ordinarily but may be 4-5 m in the heat of the day. Then they begin to circle without flapping, gaining height up to 500-2,000 m. The heavier birds are usually seen at greater heights.

From time to time they are seen *flap-gliding* or *flap-circling*, that is to say a few flaps are made during periods of gliding. This is taken as an indication that soarability is less [the ascending currents are weaker] than when there is no flapping. When soaring begins early in the day a few birds are seen flap-gliding or flap-circling; when they can soar without flapping they are joined by many others.

Cheels and vultures are seen to soar in clusters of perhaps 100-200 birds circling around the centre of the group. They move their heads about, watching the others, with no apparent effect on their flight. Presumably they centre themselves round the maximum upcurrent by noticing which of them ascends most rapidly. Occasionally part of the group breaks away and drifts to leeward. Sometimes the clusters appear as columns extended over a considerable height. [Seagulls in Britain sometimes behave similarly, though in smaller numbers.] Other less common birds, such as cranes, circle in a more disciplined group around a point outside the group.

In *flex-gliding* the wings are partially drawn in and the camber of the inner half thereby reduced. The gliding angle is increased and so it is an indication that soaring conditions are good. It is generally performed only by the birds at higher levels, indicating that soarability is greater [the ascending currents are stronger] higher up. As the heavier birds begin to circle only after the lighter birds have already been soaring for some minutes, and as they habitually remain soaring at great heights, this conclusion is strengthened, though it may partly be an indication that the soarable regions [ascending currents] are wider higher up.

Cheels are by far the most agile and acrobatic of the birds Hankin observed. They often soar in the upcurrents produced by walls, building or trees, whereas vultures usually do this only where the area of the upcurrent is more extensive – in front of a hill, for instance.

The soaring birds do not fly unless soaring of some sort is possible or they are returning to roost in the evening : they simply descend to the earth. Near Agra there was a factory for dried buffalo meat where the birds acted as scavengers, and usually if the air was at all soarable a few birds at least could be seen soaring at 500-1,000 m anywhere in the neighbourhood.

The wings of soaring birds differ from others, such as herons, owls and eagles, in that the inner part, between the elbow and 'wrist' (carpal joint) occupies nearly half the span in the former but more nearly one third in the others. A comparison can be made between cheels and eagles which are of comparable size and often encounter each other in the air : eagles appear capable of much faster flapping flight but are less efficient

* Square parentheses indicate my comments.

in soaring and are often teased by cheels which can keep up with them when the air is fully soarable. Bones of the soaring birds form a ridge on the under side of the inner part of the wings near to the leading edge, but it does not appear to have harmful aerodynamic consequences.

Aerial manoeuvres are made by rotating one or both wing tips, by increasing the dihedral angle, by moving the wings forward or backward, by flexing the wings or by retiring one or both wing tips without flexing. This last operation reduces the wing area without affecting the camber of the inner part, and the tip of the outer wing is often retired in circling : when the wing is flexed the camber of the inner part is reduced, and this is done when increased speed is required without increased lift. If the bird wishes to increase its speed quickly it rotates both wing tips downwards and undergoes a slight decrease of height [it is difficult to see how this can be easily distinguished from a single flap]. The action is called a *double dip*, and a *single dip* of one wing is used for changing direction of flight. The tail is used very little in soaring, and cheels with damaged tails are at a noticeable disadvantage only when trying to catch objects in the air or when coming to a perch.

Further discussion of the mode of flight is out of place here.

(b) *The beginning of soaring : visible heat eddies*

On a day with little or no cloud or wind, first of all heat eddies can be seen on certain walls and buildings. After a few minutes they can be seen on and over other architectural features, always in the same order as on other days. When they reach a certain intensity over a particular fort wall the air becomes soarable and within a minute 8-10 cheels can be seen circling over the city (Agra). Sometimes they change to flap-circling, in which case the heat eddies can be seen to have ceased, and soaring only begins again after the eddies have returned.

The various kinds of bird then become airborne, always in the same order, those with the largest wing loading last. When a gun is fired before the air is soarable vultures which have been started in flapping flight quickly return to the ground.

The visible heat eddies cease during the day although the air remains soarable. When thin cirrus is present the heat eddies are not observed and the air becomes soarable more gradually.

The wind is usually light early in the day and the wind that is to prevail during the day is only established sometime after the air becomes soarable for cheels. It begins in gusts, and becomes established at about the same time as the air is soarable for vultures. After this, smooth Sc clouds that have appeared in the same place during the night are replaced by cumulus clouds which move along with the wind. [The Sc clouds are presumably produced orographically, the effect being reduced as the stability of the lower layers is destroyed (Scorer 1953)]. When a wind has become established the birds begin *lee-looping*, that is circling and drifting downwind at the same time, and they can be seen returning by flex-gliding upwind from time to time. If a column of birds is drifting to leeward the upper part is usually further to leeward.

The visible heat eddies are seen at other times when cool air engulfs buildings that have become hot – in the calm after a storm that has passed quickly or in the evening or very early morning [when katabatic flow enters the edges of a town]. Those seen at dawn disappear as the sun gathers strength and the air is not soarable when they occur but only after later ones have developed as already described.

The air first becomes soarable over buildings and later over trees and gardens.

(c) *Further properties of soarable air*

Cloud shadows reduce soaring as would be expected : flex-gliding may cease although circling continues. Attempts were made to follow the motion of feathers and thistledown. Feathers do not descend in among a group of soaring birds, but may or may not elsewhere. Thistledown that is observed is usually not ascending, even though birds are soaring overhead : indeed there is no evidence of ascending air near the ground under a column of circling birds.

When the air is ascending more rapidly it is not necessary for birds to extend their wings fully. During flapping-flight the wings are slightly flexed in order to allow the feathers on the inner part of the wing to yield to the air pressure, so that during flap-gliding the wings can be seen being extended at the end of a few flaps, at the beginning of a glide. When birds are flap-gliding in search of thermals, after the passage of a cloud shadow, for instance, those at higher levels can be seen not to extend their wings fully, indicating a greater soarability [upcurrent] at higher levels.

(d) *The flight of dragonflies*

A species of dragonfly, *Pantala flavescens*, was extensively observed by Hankin. It has a wingspan of $8\frac{1}{2}$ cm and a wing loading about $\frac{1}{50}$ that of the heavier soaring birds. Much practice was required in order to discern accurately how much they flapped but it was possible to make many significant observations.

Just after sunrise or just after sunset or when there is no sunshine or wind they are usually in continuous flapping-flight. At sunset the length of glide between flapping is 10-15 cm, while in places sheltered from the wind in sunshine they may glide for 12 m or more without flapping at a height of 2-3 m above the ground.

In highly soarable air, that is in bright sunshine and almost no wind, they make turns without flapping and may follow a particular beat many times and for several minutes, it may even seem indefinitely, with no flaps. In less soarable air they give a few flaps after a turn [presumably to regain lost airspeed].

They generally begin soaring or flap-gliding earlier than cheels and stop a little earlier when the sun is obscured.

They do not circle and do not appear desirous of gaining height but merely of remaining airborne at around 2-6 m above the ground. When the air becomes more soarable they allow their abdomens and perhaps also their legs to hang down [presumably in order to act as a brake in order not to gain height or airspeed]. Thus in the middle of the day their abdomens are lowered but within less than a minute of a cloud shadow passing over them their abdomens are raised, and are lowered again within half a minute of the return of the sunshine.

When they begin to lower their abdomens those at a lower level do so first, those higher up later. At the same time soarability is increasing for birds, the heavier birds which soar higher up, taking the air later.

Some confusion was introduced into Hankin's observing for a time by the behaviour of another species (*Rhyothemis*) which possesses brightly coloured wings and usually flaps them for display in the presence of others of its species. When alone or among *Pantalae* they soar very much like them. Their flapping is more nearly 10/sec compared with the ' buzzing ' of *Pantalae* of around 30/sec.

(e) *Puffs of wind*

A puff of wind causes a few seconds lull in soarability for dragonflies and flapping flight followed by flap-gliding is temporarily necessary. This has nothing to do with

the dragonflies being blown to leeward because the puff is often only a fraction of the airspeed of the dragonflies, which are capable of rapid adjustment in ordinary circumstances. Although soarability only returns gradually for dragonflies, after a puff of wind, vultures are often seen to be circling with increased rate of ascent. The starting of a cluster of birds ascending is often coincident with a puff of wind. Then after drifting down wind for 1-5 km the birds flex-guide upwind and join later clusters at the top.

These puffs can sometimes be seen to affect only the lower branches of trees and are usually accompanied by a perceptible fall in temperature, though evidence for this is inconclusive. Low-level soarability for birds disappears soon after the puff of wind. Successive puffs of wind in a small area may be from different directions.

(f) *The passage of isolated storms*

The approach of a storm at Agra is heralded by a wind blowing towards it and against the prevailing wind which carries the storm. In this, termed by Hankin, the *attraction wind*, soarability steadily increases until many birds can be seen fast flex-gliding in different directions. Then the surface wind suddenly reverses and the temperature drops and there is much blown dust. This he calls the *displacement wind*. It is usually shallow – of the order of 70 m deep and in it soaring is not possible. This gradually decreases and there may be a shower of rain or hail. When the storm has passed overhead the displacement wind may be reversed, and decreased in strength and consequently free of blown dust; soaring is still not possible. Soarability and the prevailing wind are gradually resumed.

It is interesting to note that at Calcutta during the Monsoon, Hankin noted that the attraction wind is usually absent. Nor is it present at Agra so obviously when the the weather is widely disturbed.

(g) *Movements of birds in soaring flight*

When soaring is difficult the birds execute small, sometimes jerky, rotations about a vertical or longitudinal axis. Hankin implies that this is produced by some characteristic of the air [but it seems more likely that it consists of manoeuvres of birds in search of lift flying near stalling speed]. It is sufficient here to say that it occurs when soarability is beginning or fading away and may be taken as an indication that the birds are finding it difficult to maintain height. It occurs only at low levels, and often precedes flapping. When birds are flex-gliding it is not seen because birds are in no need to search for lift when flex-gliding occurs. It is more common when a wind is blowing, particularly in the early morning. It is not seen to occur when another movement, called *tail-jolting* by Hankin, is observed.

Tail-jolting consists of sudden forward rotations of the bird about an axis through its wingtips. It is not a manoeuvre in which the tail is used as an elevator but indicates that the angle of incidence of the wings needs to be decreased quickly.

Tail-jolting, occurring when soarability is high, more often in hot weather, is independent of the direction of flight relative to the wind, is greater at greater speeds, e.g., in flex-gliding, and is very common at the time of gusts or puffs of wind at the ground. It is very rarely observed in birds that are circling. When it is observed to be intense low down it has been observed to be followed by the occurrence of small dust devils.

Tail-jolts are usually followed by a rapid gain in height and may be preceded by double dips.

Hankin described these rotations about the three axes as ' instability.'

(h) *Observations on chimney smoke*

If smoke is disposed in horizontal layers the air is unsoarable; when it is soarable the smoke is gathered into discrete lumps.

3. The soaring flight of swallows

Since swallows were reported by glider pilots in Spain during the world gliding-contests to have been seen at 2,000 m above the ground (yet still below cloud base) it has seemed to me that a prima facie case can be made out for believing that they use thermals as their chief method of remaining airborne. A species that could soar would be at an advantage because thermals also carry up the insects upon which they feed. It is possible that they soar on their great migrations; for since a glider has covered over 800 km in a day it is reasonable to suppose that swallows could easily cover such a distance. and feed on the way. This suggestion is supported by the flight of swallows being irregular and not evidently directed like that of a pigeon.

On a few occasions I have observed swallows in the evening after all cumulus has disappeared and it is no longer possible for them to soar easily to heights at which they cannot be observed from the ground. Their height can be estimated by comparing their wing span as seen in binoculars with the length of a brick on a house at a known distance. If a single bird is watched through binoculars others will usually be seen to pass across the field of view, and it is easier to follow a group by following one of its members rather than by trying to follow the centre of the group.

On 5 July 1953, between 1800 and 1930 GMT, at Wimbledon, groups of swallows were seen flying with almost no flapping. Flapping flight of swallows consists in any case of intermittent flapping, but on this occasion flapping was almost entirely associated with manoeuvres executed in catching insects and accompanied by sudden changes in direction of flight. The insects would probably not go voluntarily to that height and if carried there by mechanical stirring would not be of sufficient concentration to attract swallows. The presence of swallows catching them therefore indicates that the air had recently ascended with little or no dilution from close to the ground. The birds seen at 200 m would not go up there if insects could equally well be caught closer to the ground and so it may be concluded that they soared there. Birds lower down, at the same time, mostly in ones and twos, were also catching insects but were having to flap in straight flight as well as when changing course and were therefore not soaring. The uppermost birds remained uppermost, which indicates that soaring, not flapping was the mode of ascent, and that they maintained height as much as possible.

The cause of the thermals on this occasion is uncertain, but was probably the heat retained in the built up area which was downwind of a large open space (Richmond Park and Wimbledon Common). There seemed to be more swallows in this locality than usual. At 1930 GMT there was considerable gusting followed by a calm during which the swallows ascended higher than before – probably to about 300 m. One bird at about 200 m was flapping a lot but when it joined two others it ceased to flap. At 1935 GMT none could be seen soaring and thereafter only solitary birds or pairs at low levels (50-100 ft) in flapping flight were seen. There was no more gusting of the wind.

It is clear that swallows, and perhaps other British soaring birds, may be able to reveal the presence of otherwise unsuspected atmospheric motion, though the prognostic value of their aerobatics may be doubted. Goodhart (1953) has described how seagulls soar in thermals but they are more often seen slope-soaring and often engage in clearly directed flapping flight. It is unlikely that the opportunity to soar in thermals is of as much consequence to them as it probably is to swallows.

4. Interpretation

An explanation of the phenomena described is now attempted. It is not our object to decide between theories but to see whether the bubble theory is adequate, and if possible to clarify it where it is obscure.

The heat is communicated in calm air to the lowest metre or two by the irregularities of the surface or by radiation. Dragonflies only soar in almost calm air because when there is a wind the heat is conveyed by mechanical stirring and useful thermals in the lowest 10 m do not exist. These thermals in calm air are hot enough to be visible to a suitably placed observer but are neither wide enough nor strong enough to support cheels which can soar close above a wall in a strong wind and can circle in a diameter of 10 m. They appear to be more of the nature of continuous columns of hot air ascending from above fixed objects. Eddies are probably not visible during the heat of the day because they are so widespread and the light so bright that they cannot be distinguished individually. No bubble structure is indicated by the flight of dragonflies in these layers. Their vertical velocity increases with increase of sunshine (dragonflies lower their abdomens).

These currents supply the wider and stronger vertical currents at higher levels for they always precede them. The bubble theory suggests that the thermals gradually penetrate to higher levels by those that are more intense leaving a wake in which further ascent is induced so long as there is an adequate supply of buoyant air. In this way the level at which soaring can take place is gradually raised. The smaller thermals including the environmental air with which they are diluted, are collected together into fewer larger ones in the wakes of those that have already penetrated farthest.

The large agglomerations form compact masses which provide a more efficient mechanism for transporting heat upwards, possibly by means of internal circulations which produce vertical velocities which are observed to be greater than in the smaller thermals. Each agglomeration only penetrates to a height which is probably about once or twice its original diameter before dilution by environmental air has affected every part of it, at which stage it becomes a preferred region for further growth into a yet bigger agglomeration by the ascent of further thermals into it. It may be necessary to regard this process as being characterized at any time by a bubble size at each height while each bubble wastes away as it ascends and leaves a wake as the embryo of a bigger bubble which will ascend further.

It is easy to understand the life history of a single bubble; but at any moment bubbles are being born anywhere that the lapse rate is superadiabatic. Thus from time to time clearly recognizable bubbles come into existence in amongst a scene of indescribable complexity. These bubbles are detectable by the discontinuities at their boundary. When one comes into existence at the ground, that is to say when a more or less compact mass of air begins to ascend from the ground its existence will be detected at the ground by the passage of a discontinuity – a puff or gust of air. This is the influx of air under a newly-formed bubble which is not observed higher up in the same way because of the wake in the lee of the bubble. The bubble can be thought of as a part of the superadiabatic layer ascending out of it, air from *above* coming down around it to take its place. The motion is beginning at this stage and resembles the motion in a convection cell to begin with, though discontinuities are quickly generated.

Thus after a puff of wind the air is unsoarable for dragonflies because the cold air spreads out over the ground and produces, temporarily a stable stratification. Meanwhile the ascending bubble offers opportunity for birds to soar.

The puffs come from all directions. There is no evidence of a continuous indraught

at the ground under rising air, but only a temporary one as the updraught begins when the bubble leaves the ground and birds begin circling in it.

The downdraughts in the passing storms, which are initiated by the precipitation, appear to act in a similar way on a larger scale and render the air unsoarable for birds. The lifting of the attraction wind over the wedge of the displacement wind provides a region of intense vertical motion in which birds can flex-glide freely. These isolated storms depend upon heat from the ground for their continued existence. The downdraughts as they spread out appear to determine from where the supply of ascending air will be taken and the storm is provided there with new life, and so appears to endure continuously. Storms in the monsoon or in widely disturbed weather depend upon a general convergence rather than surface heating for their existence, so the attraction wind, which is evidence of the growth of the thermals from the surface, is feeble. Bubbles will be more predominantly born in the layers above cloud base, as suggested by Ludlam and Scorer (1953).

Tail-jolting seems to occur when birds pass into a thermal where its boundary is well marked. Since it can be preceded by a double dip the bubble may be surrounded by a downcurrent in which the birds wish to increase their airspeed. In order to avoid being subjected to an undesirable acceleration upwards on entering the upcurrent they tip themselves forwards and so more gradually assume the upward component of the air velocity. Backward tipping is not done on entering a downcurrent but a double dip is made.

Hankin was more interested in the occurrence of these movements than in the dimensions of the air motion which made them necessary and the only definite conclusion that can be found in his observations is that when the air is highly soarable tail-jolting is more common and that it is followed by rapid ascent, though he has not reported whether the birds circle after it or not. This suggests that when the upcurrents are stronger the boundaries of the bubbles are sharper. The rotations around the vertical and longitudinal axes are most common under conditions when bubble forming near the ground is absent, namely when there is a wind and feeble sunshine. The mechanical stirring prevents the growth of a superadiabatic layer and the upcurrents are ragged and do not grow by the bubble mechanism in the stirred air. Only when the sunshine establishes a superadiabatic layer in spite of the stirring is bubble growth begun. Tail-jolting on the other hand is observed when bubble formation is most active and puffs of wind are observed at the ground. It is not observed in circling because the birds are immersed in the bubble and do not cross its boundary.

With a picture of convection in mind to act as a framework it seems that much can be learned, by observing soaring birds and dragonflies, about the nature of convection in general and at the time of observation in particular.

5. Conclusion

Besides showing that birds can provide valuable information about air movement there are repercussions for Natural Historians also.

Rainey (1951) has indicated how the desert locust in its migrating stage is dependent upon thermals in that it reacts to their presence by being carried with the wind when thermals occur but not otherwise. The small-scale motion of the air is thus seen to be as important an element in the climate in which the locust finds itself as the rainfall, temperature, or humidity. Birds like the vulture appear to depend entirely upon thermals to raise them to a place from which they can seek out carrion. Migratory birds such as storks, cranes, adjutants, and others may well depend upon thermals for their long flights,

as it has been suggested swallows do; and in the relationship between the behaviour of birds and the weather it is probable that soarability plays a larger part than any other element, and should be studied as such.

Förchtgott (1950) has indicated a lee-wave mechanism which could enable Colorado beetles to be deposited in clusters under a wave crest instead of being widely distributed by a strong wind, and movements of this scale are likely to affect the life and distribution of insects which are the meat for swallows; indeed for these purposes it may ultimately be necessary to include these movements in our description of climate.

Finally, it is hoped that this brief and inadequate discussion of a very big subject will lead meteorologists to include observations of this kind in their reports of particular phenomena, thereby increasing the detailed knowledge of them.

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Appendix

TABLE 1. Birds observed by Hankin in soaring flight

Bird	Wing span (ft)	Wing loading (lb/ft²)	Diameter of circles (m)	Time for one circle (sec)	Representative minimum ht. for flex-gliding (m)
1. Cheel (kite) (*Milvus govinda*)	4	0·55	12	7-9	80
2. Scavenger vulture (*Neophron ginginnus*)	5	0·87	—	—	120
3. Common vulture (*Pseudogyps bengalensis*)	7	1·13	40-50	13-16	400
4. Black vulture (*Otogyps calvus*)	6·5	1·23	—	—	220
5. Adjutant (*Leptoptilus dubius*)	9	1·54	—	—	800

These figures, particularly in the last 3 columns are representative only and birds' behaviour may vary widely from them on particular occasions.

Extracts from Hankin's diary

(a) *P.* 266, 11 *April* 1911

7.50 – Wind occasionally perceptible from west but leaves generally still. Smoke rising and spreading out in layers. Dust haze. Eddies on shed and cemetery. 7.55 – Heat eddies slight on barrack. 8.0 – Eddies more on barrack. 8.15 – wind coming from SW, as shown by smoke. 8.19 – slight eddies on bastion. 8.20 – one cheel circled and glided down up-wind. 8.21 – slight eddies on bastion. 8.22 – four cheels circling at low level in city, and two over Fort. Eddies stronger on bastion. 8.26 – seven cheels circling at low level, and three at high level over city.

(b) *P.* 280, 1 *April* 1912

7.30 – wind east, moving small branches in puffs. Air clear. Sekundra visible. Sky half clouded. Sunshine. No eddies on bastion. Slight eddies on cemetery. Cheels up in city and near. 7.45 – shade over city, cheels there circling as before. 8.0 – shade in city, and also a lull in the wind, as shown by smoke from a train. Cheels in city often flapping. Calm here also, and cheels near flapping or settled. 8.2 – a cheel gliding down showed digital quill gap of inside wing for steering. 8.5 – wind here. Three cheels near circling. 8.14 – wind in city, as shown by flag on fort. Cheel there circling without flapping. 8.16 – lull in city, as shown by Fort flag. Some cheels flapping.

(c) *P.* 380, 20 *October* 1912

9.10 – leaves still . . . 9.45 – Dragonflies at 2 m ht. flap-gliding. At 3 m height glides of 3 m seen. Eagle starting. 9.53 – wind now very slightly moving leaves, and perceptible. All dragonflies in sight frequently flapping. 9.58 – Calm. Dragonflies making long glides. 10.0 – calm. Glides up to 3 m length. 10.1 – glide measured 4 m length. 10.3 – wind perceptible. All dragonflies flapping. 10.8 – calm. Dragonflies making long glides and showing slight lateral instability. 10.9 – vultures beginning to circle . . . 10.20 – a puff of wind from north moving leaves. All dragonflies in constant flapping flight. A few seconds later air calm, and dragonflies a few metres to leeward making long glides.

551.577.2 (42)

Rainfall in the Isle of Man

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SUMMARY

The rainfall of the Isle of Man has been examined, making use of all available data, much of it relating to only very short periods. This discussion has been made possible by treating all the short records from the aspect of their differences on Douglas. The Island is divided into seven rainfall zones, the characteristics of which are discussed. Maps of the average annual fall and the average number of rain-days and wet-days are presented. Fluctuations of annual rainfall have been found to lie between 64 and 145 per cent of average. When compared with other Irish Sea coastal areas, the Douglas average rainfall is shown to be most closely correlated with that of the North Wales coast and South Scotland.

1. INTRODUCTION

One difficulty besetting climatologists is the representation of a mass of meteorological observations in an easily digestible form. In an attempt to provide an answer to this problem, the following paper suggests a meteorological analogy of the tidal data as presented in *Admiralty Tide Tables*, by which method the data for a standard port, chosen almost invariably where there is a wealth of reliable observations, are listed in detail, and the data for neighbouring and less important places are given as differences on the standard port.

The rainfall of the Isle of Man is suitable for such a review, having one good long-period series of observations, and many other records which are of themselves of too short duration to yield anything of value except by some weighting method. The size

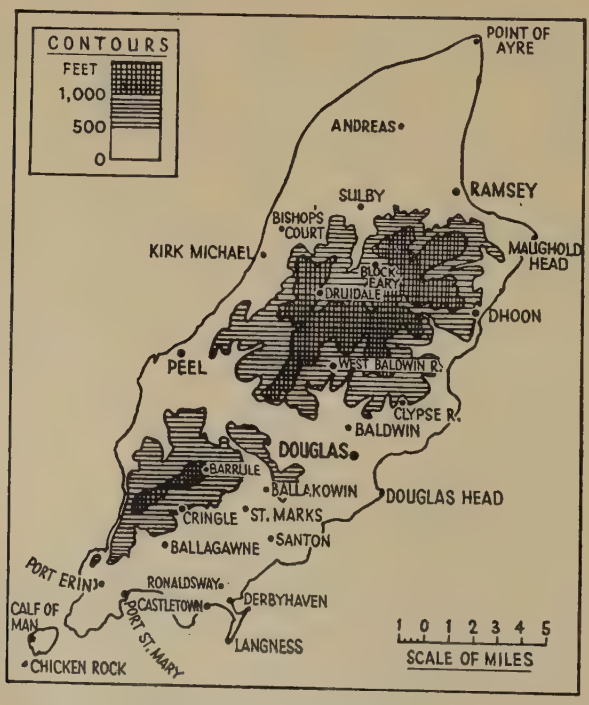

Figure 1. Index map of stations.

of the Island (33 mi by 12 mi) and its situation indicate that all parts should be subject to the same rainfall regime, modified only by orographic and other purely local topographical effects. The local climate has also been only very slightly documented in meteorological literature.

The data examined, mainly extracted from *British Rainfall* and the *Monthly Weather Report*, cover the period 1871-1952, as prior to 1871 there were rarely, if ever, more than three gauges in use simultaneously and some of these were in over-exposed places such as lighthouses. Records were obtained from 36 stations and the results are given in Tables 1 and 2. In a few cases the station names have been slightly changed from the form given in *British Rainfall*, usually for reasons of clarity and precision. For example, reference to Fig. 1 (showing the location of the stations and general relief) will indicate that such a description as Douglas (Cringle Reservoir) is misleading.

2. Methods of reduction

The first step was the compilation of records for a standard place throughout the whole period, with which all the shorter records could be compared. Douglas was the obvious choice, not only because it is the main centre of population, but because records exist locally throughout the 82 years, although not from one station. Borough Cemetery carries the record back to 1909, and Cronkbourne from that date a further 29 years. The records from these stations are combined in the Meteorological Office *Book of Normals*, and this excellent precedent has been followed. Prior to the first record for Cronkbourne in 1880, Derby Square and Victoria Road catches were meaned for 1875-9, and for 1871-4 the Derby Square total was accepted. The means computed for these stations were found in the main to have no significant difference statistically amongst themselves, but Derby Square does exhibit signs of being significantly drier than Borough Cemetery, so it is fortunate that only four years' means are drawn exclusively from the former. Based on these calculations the Douglas mean annual rainfall for the period 1871-1952 is 43·50 in. The variation in each year is shown in Fig. 2, each year's fall being expressed as a percentage of the average.

For stations other than Douglas, the arithmetic mean, if derived from over 20 years' records, was deemed to be close enough to the long period average to be accepted. Stations whose records were between 10 and 20 years' duration had their long-period average computed by correcting the arithmetic mean by the ratio which the mean of the same years of the Douglas record bore to its long-period average. When the record was of shorter duration than ten years, a linear regression equation was computed between it

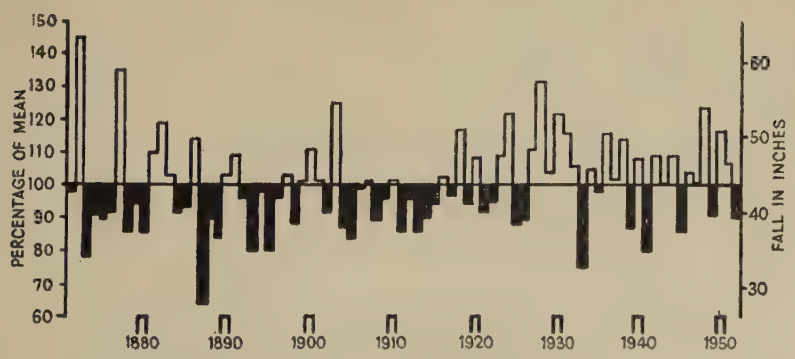

Figure 2. Douglas annual rainfall, 1871-1952.

and Douglas, and the average determined by applying the constants so calculated to the Douglas average. In these calculations the correlation coefficients between Douglas and the various records were very high. In 11 cases the coefficient was over 0·90, and in only three cases was it less than 0·80. The lowest value was 0·69 at Block Eary. This close correlation was taken to be strong confirmation for the view put forward earlier that all parts of the Isle of Man are subject to the same rainfall regime, and the local variations are due only to topographical modifications of this regime.

The catch at Douglas Head showed little correlation with Douglas, and appears much too small, so it was discarded from consideration. The rainfall totals at Port Erin and Druidale, where only two years' records existed, and the corresponding Douglas values, were applied to simultaneous equations, and the solutions used to give a best estimate of the local average with reference to the Douglas 82-year average.

It is fully realized that some of the foregoing method leaves much to be desired, but it is the author's wish to extract the maximum information from the somewhat meagre data available.

TABLE 1. SUMMARY OF RECORDS OF LONGER THAN 20 YEARS' DURATION

Place	Mean rainfall (in.)	Standard deviation (in.)	Standard error of mean (in.)	Maximum fall amount (in.)	Maximum fall year	Minimum fall amount (in.)	Minimum fall year	No. of years of record	Ht. above sea level ft
Langness	30·26	4·38	0·64	41·84	1928	19·10	1885	47	75
Derbyhaven	33·21	5·34	1·11	44·15	1910	20·64	1893	23	20
Santon	36·22	5·54	0·94	48·11	1924	24·13	1904	35	250
DOUGLAS	43·50	6·30	0·70	62·94	1872	27·64	1887	82	(210)
Douglas (Cronkbourne)	41·99	5·40	1·00	51·61	1882	27·64	1887	37	137
Douglas (Derby Sq.)	39·01	7·92	1·58	62·94	1872	24·81	1887	25	88
Douglas (Victoria Road)	41·77	6·28	1·30	58·66	1877	26·98	1887	23	120
Douglas (Borough Cem.)	44·43	5·33	0·81	57·38	1928	32·76	1933	43	284
Clypse Reservoir (a)	44·92	5·71	0·70	55·26	1928	27·99	1887	65	439
Clypse Reservoir (b)	50·83	5·82	0·72	61·47	1928	33·66	1887	65	439
Bishop's Court	36·36	4·50	0·96	48·54	1903	30·03	1893	22	(125)
Maughold Head	34·34	7·85	1·31	52·63	1928	15·18	1917	36	230
Ramsey (Town Hall)	43·88	7·40	1·04	67·43	1928	29·27	1906	51	(25)
Ramsey (North Prom.)	46·21	7·23	1·45	65·35	1928	29·99	1933	25	5
Point of Ayre (a)	27·14	4·83	0·68	42·83	1872	17·18	1887	50	27
West Baldwin Res.	60·23	7·39	1·37	78·30	1928	43·16	1933	29	480

Heights quoted in brackets have been estimated

TABLE 2. RECORDS OF SHORTER DURATION THAN 20 YEARS

Place	Mean rainfall (in.)	Ht. above sea level (ft)	Place	Mean rainfall (in.)	Ht. above sea level (ft)
Chicken Rock	22·95	126	Douglas Head	27·94	—
Calf of Man	27·53	325	Peel	36·33	210
Port Erin	30·93	—	Baldwin	54·91	120
Port St. Mary	26·65	40	Dhoon	54·50	450
Castletown	32·73	30	Druidale	55·13	1,000
Ronaldsway	33·18	55	Block Eary	58·18	620
Ballagawne	38·67	365	Kirk Michael	36·28	100
Cringle	47·63	500	Sulby	43·79	106
St. Marks	37·70	400	Ramsey	43·31	30
South Barrule	56·87	1,000	Andreas	34·50	72
Ballakowin	43·68	374	Point of Ayre (b)	32·94	27

The standard deviation (S.D.) and standard error of the mean (S.E.) of the longest records appear in Table 1. Differences of mean annual rainfall cannot be considered significant unless the means differ by three times the S.E. of the difference. As examples of the magnitudes obtained the lowest S.E. of the difference was 0·92 in. and the highest was 2·00 in. So differences of less than 3 in. can hardly be considered significant. Because of the large S.E. involved, rainfall will only be quoted to the nearest inch when considering the various rainfall zones of the Island.

3. Rainfall zones

Figure 3 shows isohyets of the mean annual rainfall. These have been drawn with due regard to both the station averages and the contours. It is not suggested that the map must be accepted absolutely, and *in toto*, but it presents the data in a graphical form and some inference has to be made to cover parts of the Island where no gauges have been installed. In an analysis utilizing all the records it was not possible to trace any close correlation between rainfall and altitude, but when only the records from the area south of the Douglas-Peel line were analysed, a linear regression equation emerged quoting a sea-level rainfall of 31·0 in., which increased with altitude at a rate of 2·7 in. per 100 ft. Extending to the whole of the Island, this formula expesses the rainfall of nine stations within five per cent, and of another seven stations within ten per cent. It does not express adequately the rainfall of the Douglas and Ramsey stations or the Mountain group except Druidale (within six per cent).

The Isle of Man may be conveniently divided into seven rainfall zones.

(a) The southern plain with a mean rainfall of the order of 33 in., with a lower fall of 30 in. at the more exposed Langness, and, rather doubtfully, also at Port Erin (based on two years only). A mean of 27 in. around Port St. Mary appears possible but this average has been computed from five years of a gauge mounted 3 ft above ground. It was

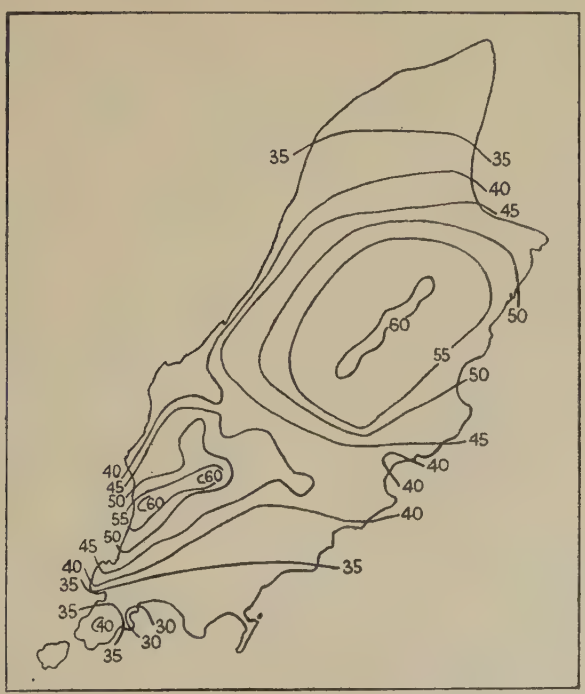

Figure 3. Mean annual rainfall map of the Isle of Man.

examined statistically by means of Student's t distribution, and only reached the five per cent level of probability as being significantly drier than the Castletown group of stations.

An inspection of the site of this gauge makes one all the more conscious of the need for caution before accepting its records. A new and admirable gauge has, however, been installed at Port St. Mary in recent months and when sufficient records are available from it, a better estimate of the true local rainfall can be made.

(b) There is a steady rise in the catch northwards from zone (a) as the altitude increases via Santon (36 in.) and St. Marks (38 in.), and this increase continues over the crest of the range of low hills with Ballakowin, and in the central valley, Douglas (both 44 in.).

(c) Zones (a) and (b) are flanked on the west and NW. by a range of hills with summits varying from 766 ft in the south to 1,584 ft at the northern end. On the eastern slopes three gauges, Ballagawne (365 ft), Cringle (500 ft), and South Barrule (1,000 ft) give average catches of 39 in., 48 in. and 57 in. respectively. No records exist from the precipitous western slopes of this range and rainfall totals are unknown. In view of the sheer cliffs rising for hundreds of feet straight out of the sea from Bradda Head almost to the Niarbyl, with only one narrow gap less than $\frac{1}{4}$ mile wide, no reliable records could ever be obtained here, and for this small area, the isohyets of Fig. 3 are very conjectural.

The small areas around South Barrule and Cronk ny Irree Laa, with annual rainfall greater than 60 in. shown in Fig. 3, are also an assumption of the author, but in view of 57 in. at 1,000 ft locally, they seem quite probable.

(d) Still further north lies the mountain block rising to 2,034 ft at Snaefell. No gauge has been established high up in this massif, the highest at Druidale (1,000 ft) only operated for two years, and knowledge is restricted to gauge records in the steep-sided valleys for an estimate of the regional rainfall. The highest mean is 60 in. at West Baldwin Reservoir. Two gauges were read here simultaneously for many years, and the results provide us with an interesting study in exposure. Records from No. 1 gauge were available from 1922 to 1942, and for No. 2 gauge from 1929 onwards. The two gauges were located at a similar height and close together, but on opposite sides of a valley, No. 2 being on the western side and sheltered from the prevailing SW. wind, while No. 1 gauge was exposed to the full sweep of such winds. The difference in the annual catches varied from 4 to 9 in., the more exposed gauge always recording less rain. The two sets of readings for the ten years 1933-1942 were analysed and the following regression equation expressed the fall in inches at No. 2 gauge (R_2) in terms of the fall at No. 1 gauge (R_1).

$$R_2 = 1{\cdot}02\,R_1 + 5{\cdot}11 \quad . \quad . \quad . \quad . \quad (1)$$

With these discrepancies in mind, the Meteorological Office advised the discontinuance of No. 1 gauge in 1941, and its records ceased soon afterwards.

The exposure of these two gauges has been examined in detail partly because of the bearing it may have on the mean recorded at Clypse Reservoir (45 in.), the lowest average fall by almost 10 in. in the whole of the mountainous area. This gauge is located in a valley facing the SW. winds and its exposure suffers from the same faults as No. 1 gauge at West Baldwin Reservoir. If we apply the same correction to its readings we obtain a mean rainfall for the locality of 51 in., a figure in fair agreement with that recorded at Baldwin (55 in.) less than three miles away. Table 1 shows both the original and adjusted data for Clypse Reservoir as (a) and (b) respectively.

The other records from this region are of short duration, and beyond noting that they are all in close conformity with each other, giving annual falls between 55 and 58 in., little comment can be made. In this zone the rainfall does not increase regularly with

altitude and the only pointer towards a reason for the local variations is the fact that the two heaviest falls are recorded at sheltered gauges in deep valleys, at West Baldwin Reservoir and Block Eary.

(e) The narrow west coastal strip north of Peel has a mean annual rainfall of 36 in.

(f) The flat alluvial plain north of Sulby and Ramsey shows a decreasing fall northwards from 36 in. at Bishop's Court in its SW. corner, to 34 in. at Andreas in the centre, and to 33 in. at Point of Ayre, the extreme northerly tip of the island. From this station, two distinct series of readings are available :

(i) covering the period 1871-1921 and having a mean of 27·14 in. and a standard deviation of 4·83 in.

(ii) relating to 1935-51, with a mean of 34·01 in., and a standard deviation of 3·83 in.

Confirmation that those two records must be treated separately is obtained on considering the average number of rain-days annually, as they have means of 141 and 191 days respectively. It would appear that station (a) suffered from over-exposure, the usual fault of Manx lighthouse gauges, and its records have been disregarded when drawing isohyets. For this purpose the arithmetic mean of station (b) has been corrected to a comparable basis with the Douglas long-period mean and reads 32·94 in.

(g) Between this flat plain and the mountains lies a narrow belt from Sulby to Ramsey with the heaviest rainfall of any part of the Isle of Man at heights around sea level. The annual fall varies between 43 and 46 in.

The foregoing review has omitted all reference to four of the available records. Three of these, Calf of Man, Douglas Head, and Maughold Head are from gauges situated in a very turbulent position on cliff tops. The fourth, the Chicken Rock gauge, was on a lighthouse built on a half-tide rock. All these records give annual means which are distinctly low when compared with neighbouring observations which are normally exposed. It is considered that they should be rejected owing to their unorthodox surroundings. This is doubly a pity as a long record of 36 years must be ignored, and it deprives us of all knowledge of the rainfall in the extreme SW., an area of great interest owing to its varied terrain within short distances.

Comparison is inevitable between Fig. 3 and the *Average annual rainfall map* published by the Ordnance Survey from information supplied by the Meteorological Office. It should be borne in mind that the latter is weighted so as to relate to the period 1881-1915, whilst Fig. 3 refers to 1871-1952 and is the nearest approach which may yet be given to a timeless mean.

In general the isohyets are very similar in the two maps, although Fig. 3 shows a generally higher rainfall, with greater variability. The higher fall is only to be expected as it is universally agreed that the period 1881-1915 gave a slightly lower rainfall over the British Isles than the years before and after it.

Computation from Fig. 3 shows that a very small area of the Isle of Man (less than $\frac{1}{2}$ mi^2) receives less than 30 in. of rain annually, 80 mi^2 receive 30-40 in., 72 mi^2 receive 40-50 in., 65 mi^2 receive 50-60 in. and 4 mi^2 receive just over 60 in. These areas total 221 mi^2, which is the area of the Isle of Man quoted in the *Oxford Atlas*.

4. Annual rainfall fluctuations

Table 3 shows the wettest and driest groups of years at Douglas. As is general throughout the British Isles, 1872 was the wettest year. 1887 was considerably drier than either 1921 or 1933. The period 1927-32 was the only occurrence of six consecutive years each with a fall above average. The maximum number of consecutive years each with falls below average is five, this occurring twice, 1892-6 and 1911-5. There is an

appearance of periodic variation in Fig. 2 with maxima around 1872 and 1928, and minima around 1904 and 1942, but there is nothing to suggest that even if this periodicity is real it will be maintained.

The heaviest annual fall recorded in the Isle of Man was 71·75 in. at West Baldwin Reservoir No. 1 Gauge in 1928. Correcting for the over-exposure of this gauge (see Section 3), increases this total to 78·30 in., 130 per cent of average. This is a very close approximation to the 132 per cent of average rainfall which fell at Douglas in the same year. From this it appears likely that in 1872 a fall of the order of 87 in. (145 per cent of average) would have occurred at West Baldwin Reservoir, and probably this may be treated in practice as the maximum possible rainfall in the Isle of Man. In confirmation of this, the Douglas total for 1872 departs from the mean by almost exactly three times its S.D.

Turning to the driest possible year it becomes difficult to evaluate an absolute minimum owing to the number of over-exposed gauges. The lowest percentage fall at Douglas is 64. At Langness, 19·18 in. (63 per cent of average) was measured in 1885, and the same percentage at Point of Ayre (a) in 1887. The fall of 15·18 in. at Maughold Head in 1917 is very suspect, as indeed is the whole record there. Not only is the mean very low as compared with nearby gauges but the S.D. is abnormally high compared with the mean, this indicating some instability in the record.

The frequency distribution of Douglas rainfall is almost normal, being slightly positively skew, 43 years having below the average fall, and 39 years a fall above average. At the lower end of the distribution, one year shows a mean departure greater than twice the S.D. At the upper end three years show departures from mean greater than 2 × S.D. and one year greater than 3 × S.D. The frequency distributions at most of the other stations listed in Table 1 are a very close approximation to normal. Bishop's Court was markedly positively skew. As the Point of Ayre (a) and Maughold Head records have already been rejected, their frequency distributions were not investigated.

In so far as the above observations relating to extreme years cover ground previously examined (Glasspoole 1926) they show that the earlier figures have not yet been superseded.

TABLE 3. WETTEST AND DRIEST CONSECUTIVE YEARS AT DOUGLAS

No. of years	Period	Mean annual rainfall (in.)	Percentage of 82-year mean
(a) wettest			
1	1872	62·94	145
2	1927-28	52·91	122
3	1928-30	51·92	119
4	1928-31	51·59	119
5	1927-31	50·96	117
6	1927-32	50·16	115
(b) driest			
1	1887	27·64	64
2	1887-88	33·46	77
3	1887-89	34·51	79
4	1887-90	37·10	86
5	1892-96	38·98	90
6	1893-98	39·55	91

5. RAIN-DAYS AND WET-DAYS

Figs. 4 and 5 show the distribution of the average number of raindays and wet-days respectively. A rain-day is defined as a period of 24 hr commencing at 09 GMT during

which 0·01 in. or more of rain is recorded. A wet-day is a similar period during which 0·04 in. or more of rain is measured. Both these maps follow the same general shape of Fig. 3. The mountainous area of the north has the most rain-days as might be expected and the SW. has the fewest. On considering the wet-days, it seems that the hills of the south receive 0·04 in. more frequently than the mountain block, but this is based solely on the testimony of one gauge. The average number of rain-days observed at Maughold Head is in reasonable agreement with neighbouring gauges, and has therefore been plotted. The number of wet-days here is in still better agreement, suggesting that the sparsity of rainfall is due to a substantial loss on days of heavy rain, perhaps accompanied by high wind.

The average difference between the station averages of rain-days and wet-days is 36. Several stations notably Ramsey (Town Hall) (54), Clypse Reservoir (55), Point of Ayre (b) (49), Port St. Mary (19), Langness (18), and Maughold Head (17) differ markedly from this average value.

The increase in the number of wet-days, from Langness with the fewest to Cringle with the most, is 31 per cent. By way of comparison, the increase in average rainfall from Langness to Cringle is 58 per cent, and from Langness to the wettest station almost 100 per cent. The increase in the number of rain-days from the station with the lowest frequency to that with the highest is 32 per cent.

The S.D. of the number of rain-days at Douglas is 18·1, and the standard error of the mean is 2·1. For wet-days the Douglas S.D. is 15·6 and the S.E. 2·8. Assuming these figures to be approximately applicable to the values for other stations, this supports the view that a difference of 12 days is needed between the average number of wet-days at any two stations for them to be significantly different. So even if the sparsity of observations did not prevent the drawing of isopleths on Figs. 4 and 5 at closer intervals than 10 days apart, they would be statistically unreliable.

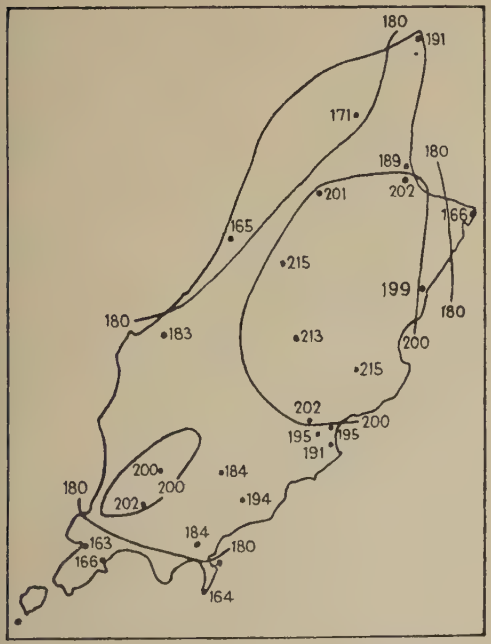

Figure 4. Isopleths of annual rain-day averages.

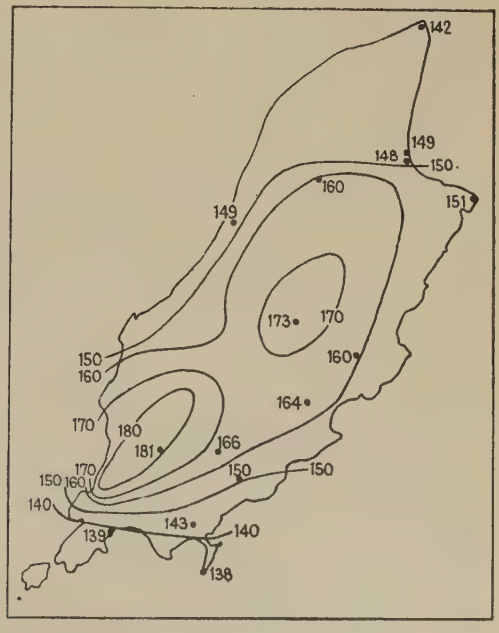

Figure 5. Isopleths of annual wet-day averages.

6. Monthly rainfall

The officially published data on Manx monthly rainfall relate almost exclusively to Douglas. 17½ years of Point of Ayre (b) data and 5 years of Ronaldsway data have also been printed in the *Monthly Weather Report*. With the meagreness of these data in mind, we will examine first in detail the Douglas record (see Table 4), and then consider what modifications are required to apply this to the remaining parts of the Island, and what accuracy may be expected when this is done.

TABLE 4. Douglas monthly rainfall (in.)

Month	Mean rainfall 1881-1952	1881-1915 (a)	1916-1950 (b)	Standard deviation	(b) − (a)	Wettest month Amount	Year	Driest month Amount	Year
Jan.	4·22	3·35	5·07	1·81	+ 1·72	10·89	1948	0·83	1880
Feb.	3·13	3·19	3·13	1·68	− 0·06	7·61	1937	0·15	1932
Mar.	2·95	2·96	2·90	1·44	− 0·06	7·84	1947	0·89	1944
Apr.	2·50	2·44	2·62	1·04	+ 0·18	5·04	1899	0·41	1938
May	2·71	2·50	2·90	1·34	+ 0·40	6·57	1886	0·24	1896
June	2·58	2·42	2·79	1·69	+ 0·38	6·44	1928	0·11	1925
July	3·03	3·06	3·08	1·21	+ 0·02	6·57	1888	0·28	1898
Aug.	3·88	3·81	3·88	1·85	+ 0·07	7·68	1891	0·31	1947
Sept.	3·77	3·27	4·21	1·80	+ 0·94	10·38	1950	0·47	1894
Oct.	4·75	4·54	5·07	2·12	+ 0·53	9·18	1903	0·54	1946
Nov.	4·74	4·71	4·73	2·19	+ 0·02	11·08	1931	0·27	1945
Dec.	4·98	4·94	4·96	1·86	+ 0·02	11·31	1934	1·24	1926
Year	43·94	41·19	45·34	6·30	+ 4·15				

The average monthly rainfall for the two 35-year periods, 1881-1915 and 1916-1950, (the former extracted from already published sources) is shown separately in Table 4. There is very close agreement between these two periods, the only months with appreciable differences being January and September which are both wetter in the later period. The annual average for the earlier period is 41·19 in. and for the later period 45·34 in., an increase of 4·15 in. The statistical significance of these variations was tested with reference to Student's t distribution. The value for t for January was 3·91, for September 2·18, and for the whole year, 2·72. Entering tables of t distribution with 68 degrees of freedom in each case, the chances of such differences in means being observed without there being a real difference in the rainfall are less than 0·001, between 0·05 and 0·01, and less than 0·01 respectively. None of the other differences is statistically significant.

In the 1881-1915 period, December is the wettest month and April, May and June share the honour of being the driest months. There is a subsidiary minimum in September; apart from this the months from June to December become steadily wetter. January is over 1½ in. drier than December, and allowing for its smaller number of days, February is slightly wetter than January.

In the 1916-1950 period, October, November, December and January are almost equally wet, and March, April, May, June and July almost equally dry. February, August and September are the transitional months between these extreme seasons.

The variability of monthly rainfall may be deduced from the S.D.'s and extreme values shown in Table 4. June is the month of greatest percentage variability (S.D. equals 65 per cent of average fall), and December that of the smallest percentage variability (S.D. equals 37 per cent of average fall).

In view of the shorter lengths of the Point of Ayre (b) and Ronaldsway records, very little can be derived from them directly, but their salient points are summarized in Table 5. Two forms of the Ronaldsway monthly means are quoted; (a) is the uncorrected arithmetic mean of the five years of observation and (b) has been weighted to convert it to the 35-year period, 1916-1950.

The substantial differences between columns (a) and (b) in at least nine months are justification for a correction such as this, and no great reliance can be placed on values derived from short periods of observations unless weighting methods are adopted.

TABLE 5. POINT OF AYRE (B) AND RONALDSWAY MONTHLY RAINFALL

Month	Point of Ayre (b)				Ronaldsway mean	
	Mean	Standard deviation	Wettest month	Driest month	(a)	(b)
Jan.	3·95	1·54	8·44	1·56	3·94	3·88
Feb.	2·20	1·00	4·40	0·75	2·00	2·31
Mar.	2·36	1·32	6·38	0·50	2·49	2·00
Apr.	1·82	0·60	3·82	0·37	1·86	1·87
May	2·21	1·18	5·34	0·40	1·62	2·04
June	1·97	1·15	3·98	0·29	1·50	1·88
July	2·41	0·76	3·78	1·02	2·40	2·69
Aug.	2·80	1·62	6·41	0·15	3·58	2·58
Sept.	3·44	1·29	5·86	1·53	4·03	3·02
Oct.	3·58	1·70	6·06	0·39	3·10	3·81
Nov.	3·75	1·70	6·69	0·28	3·68	3·47
Dec.	3·23	1·21	5·77	1·06	3·09	3·63

When attempting comparison between these short records and Douglas, the best approach is by way of correlation with the simultaneous Douglas record. The correlation coefficient (r) between the individual monthly totals of the Ronaldsway and Douglas falls for 1948-1952 is 0·92. Between the individual monthly totals for 1935-1952 observed at Point of Ayre (b) and Douglas, r equals 0·87. In spite of these high correlations, attempts to estimate the Ronaldsway rainfall of any month selected at random by computation from the Douglas catch are liable to a S.E. of 0·62 in., so that the error of an estimate thus made will be greater than 0·42 in. on 50 per cent of occasions, greater than 1 in. on 10 per cent of occasions and greater than 1·5 in. on 2·5 per cent of occasions. Similar computations for Point of Ayre (b) show a S.E. of 0·74 in.

The relevant regression equations are :

$$\text{(a) for Ronaldsway, } r_x = 0{\cdot}69\, r_d + 0{\cdot}18 \quad . \quad . \quad . \quad (2)$$

$$\text{(b) for Point of Ayre (b), } r_x = 0{\cdot}66\, r_d + 0{\cdot}01 \quad . \quad . \quad (3)$$

where r_x is the rainfall to be computed and r_d the fall at Douglas in the same month.

The constant in Eq. (3) is small enough to be neglected in practice.

The monthly falls at Point of Ayre (b) approximate to the normal frequency distribution, but extreme values are rather more frequent than one would expect. The records from Ronaldsway are rather too short to make even the most cursory statistical analysis worth while.

7. CORRELATION WITH OTHER IRISH SEA GAUGES

Correlation coefficients were evaluated between the mean monthly rainfall at Douglas for the period 1881-1915 and similar statistics for 25 other stations bordering on the Irish Sea, and the results are plotted in Fig. 6. All the gauges selected were close to the

Figure 6. Correlation coefficients between Douglas and other Irish Sea monthly rainfalls.

coast, and at an altitude of not more than 300 ft above Ordnance Datum. Manx rainfall is most closely allied to the rainfall of the North Wales coast, extreme south Scotland and the tip of Cumberland. On the edges of these zones, the close correlation falls off. In the areas of low correlation, this is due to relatively lower spring rainfall and relatively higher August rainfall than in the Isle of Man.

8. Conclusions

The annual rainfall of the Isle of Man divides conveniently into seven zones, and sufficient data have been accumulated to estimate reliably the average annual fall of all parts with the exception of the extreme SW., the western slopes of the southern hills and land over 1,000 ft in height. The extreme annual fluctuations of rainfall vary from 64 to 145 per cent of average.

The maps showing the annual distributions of rain-days and wet-days are very similar to the isohyetal map, but with a much smaller variation.

Lengthy monthly statistics are only available for Douglas and though the correlation between this record and stations in both the north and south of the Island is high, attempts to estimate local monthly rainfall from the Douglas record are liable to appreciable errors.

The Manx rainfall regime correlates more closely with that of the coastal zones immediately north and south of it rather than with either the English or Irish coastal rainfall.

Reference

Glasspoole, J. . . . 1926 *Quart. J. R. Met. Soc.*, **52**, p. 237.

551.574.1

Dimensionless equations for the growth of a drop of salt solution

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SUMMARY

The usual equations for the transfer of heat and water vapour to and from a drop of salt solution in an atmosphere with which the drop is not in equilibrium involve the relative humidity of the atmosphere, the drop size, the mass of the salt nucleus and various physical characteristics of air and water which depend upon temperature. Generally an explicit solution of the equations is not possible and, if more than one set of circumstances are to be considered, considerable labour may be involved in obtaining numerical solutions. The following paragraphs indicate a convenient way of manipulating the equations so that the effects of different temperatures and different salt nucleus masses may be allowed for by means of a simple multiplying factor.

1. Equilibrium vapour pressure over a hygroscopic droplet

Wright (1936) has given an equation for the vapour pressure in equilibrium with a drop of Na Cl solution; this equation may be written in the form

$$H = \exp\left(\frac{P}{2a}\right) - \frac{m}{6a^3} \qquad (1)$$

where

$100\,H$ = relative humidity per cent in equilibrium with the drop.
a = drop radius (cm).
m = mass of Na Cl nucleus (g).
$P = 4TM/\rho_1 R\theta$.
T = surface tension (74 dynes cm^{-1} for water).
M = molecular weight of water (18).
ρ_1 = drop density (g cm^{-3}), assumed to be unity.
R = gas constant ($8{\cdot}314 \times 10^7$ erg deg^{-1} mol^{-1}).
θ = temperature (°A).

Putting numerical values for the various parameters we find $P = 6{\cdot}408 \times 10^{-5}/\theta$.

In passing we may note that the factor $\frac{1}{6}$ in the last term on the right hand side of Eq. (1) is the arithmetical result of inserting numerical values in a function which depends upon the physical properties of Na Cl. As Wright shows, the factor becomes $\frac{1}{10}$ if the nucleus consists of sulphuric acid but otherwise Eq. (1) is unaltered.

By differentiating Eq. (1) with respect to a and replacing the exponential term in the result by unity (since $P/2a$ is normally less than 2×10^{-3}) we find that H has a maximum value H_c when a has a critical value of a_c where

$$a_c^2 = \frac{m}{P} \qquad (2)$$

If we expand the exponential term in Eq. (1), neglect the second and higher powers of $P/2a$ (we justify this below), and write h for the supersaturation $(H - 1)$ we get

$$h = \frac{P}{2a} - \frac{m}{6a^3} \qquad (3)$$

and substituting a_c for a in this equation we find that the value of h corresponding to a_c is given by

$$h_c = \frac{P}{3a_c} \qquad (4)$$

If, now, we change the variables in Eq. (3) by means of the substitutions

$$a = xa_c, \quad h = yh_c \quad . \quad . \quad . \quad . \quad . \quad (5)$$

we get

$$\frac{y}{3} = \frac{1}{2x} - \frac{1}{6x^3} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Eq. (6) is independent of temperature and of nucleus mass and represents a relation between drop radius (expressed as a multiple of a_c) and the supersaturation in equilibrium with the drop (expressed as a multiple of h_c). The relationship is shown in Fig. 1. This single curve, together with Eqs. (2) and (4), is equivalent to the family of curves sometimes used elsewhere (see, e.g., Simpson 1941).

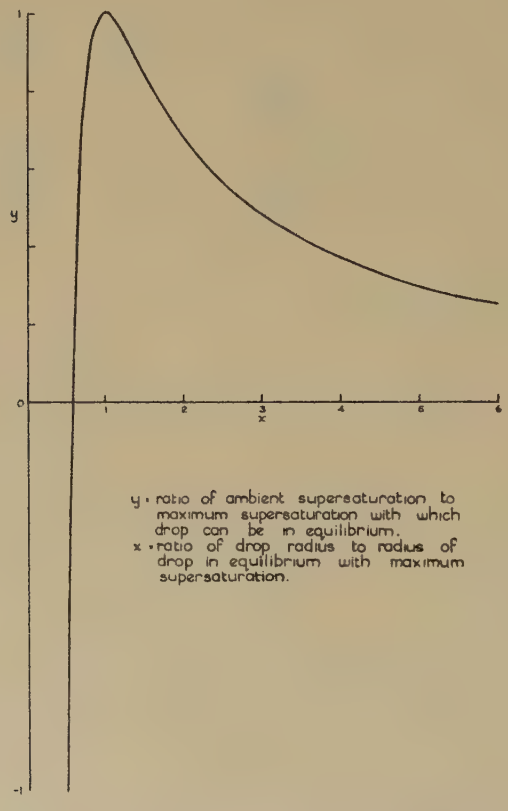

Figure 1. Stability curve for drop of salt solution.

When deducing Eq. (3) we neglected second and higher powers of $P/2a$ in comparison with $[(P/2a) - (m/6a^3)]$. It is easily seen that this is equivalent to assuming that $P^{3/2}/2m^{\frac{1}{2}}$ can be neglected in comparison with $|x - (1/3x)|$. Now P is of the order 2×10^{-7} and m is usually greater than 10^{-16} so that $P^{3/2}/2m^{\frac{1}{2}}$ is less than 5×10^{-3} and it is easily demonstrated that the numerical value of $(x - 1/3x)$ exceeds 5×10^{-3} by a factor of 5 or more except in the neighbourhood of $x = 3^{-\frac{1}{2}}$. But when $x = 3^{-\frac{1}{2}}$ we get, from Eq. (1), $h = \exp(b) - b - 1$ where $b = P^{3/2}\,3^{\frac{1}{2}}/2m^{\frac{1}{2}}$. Thus b is less than 10^{-2} and the error in assuming h to be zero, as indicated by Eq. (3), is seen from tables of the exponential function to be less than 5×10^{-5}. With the values of P and m quoted above h_c is $4{\cdot}5 \times 10^{-3}$. We may accordingly regard the assumptions involved in Eq. (3) as justified.

2. Rate of growth of a drop of salt solution

Now consider a drop of salt solution surrounded by an atmosphere containing water vapour and suppose that the vapour pressure at the surface of the drop is not equal to the ambient vapour pressure. Vapour is transferred between the drop and the atmosphere and owing to the release or absorption of latent heat of condensation the temperature at the drop surface differs from the ambient temperature. The rate of transfer of heat across any spherical surface of radius r is $4\pi r^2 k\, d\theta/dr$ where θ is the ambient temperature and k is the thermal conductivity of air. This rate must be the same for all such surfaces and must also equal the rate of release or absorption of latent heat at the drop surface. These considerations easily lead to

$$\Delta\theta = \frac{\lambda}{kM} a \frac{da}{dt} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where

λ = latent heat of vaporization of water (cal mol^{-1}).

k = thermal conductivity of air (cal cm^{-1} sec^{-1} deg^{-1}).

$\Delta\theta$ = excess of drop-surface temperature over ambient temperature at a great distance (°C).

t = time (sec).

The drop density is assumed to be unity. Similar treatment of the rate of transfer of water vapour across a spherical surface leads to the equation

$$p_\infty - p_a = \frac{R\theta}{DM} a \frac{da}{dt} \quad . \quad . \quad . \quad . \quad (8)$$

where

D = coefficient of diffusion of water vapour in air (cm^2 sec^{-1}).

p_∞ = vapour pressure at a great distance (dynes cm^{-2}).

p_a = vapour pressure at the drop surface (dynes cm^{-2}).

If p_θ represents the saturation vapour pressure at a temperature θ we may write

$$p_\infty = Hp_\theta, \quad p_a = \left(p_\theta + \Delta\theta \frac{dp_\theta}{d\theta}\right)\left\{\exp\left(\frac{P}{2a}\right) - \frac{m}{6a^3}\right\} \quad . \quad . \quad (9)$$

Now if from Eqs. (7), (8) and (9) we eliminate $\Delta\theta$, p_∞ and p_a, expand the exponential term in Eq. (9) and neglect second and higher powers of $P/2a$ and also change the variables from a and H to x and y by means of Eqs. (5), we obtain

$$x\frac{dx}{dt} = \frac{\dfrac{MD\,p_\theta\,P}{R\theta\,a_c^3}\left[\dfrac{y}{3} - \dfrac{1}{2x} + \dfrac{1}{6x^3}\right]}{1 + A + A\dfrac{P}{a_c}\left(\dfrac{1}{2x} - \dfrac{1}{6x^3}\right)} \quad . \quad . \quad . \quad (10)$$

where

$$A = \frac{\lambda D}{R\theta k}\frac{dp_\theta}{d\theta}$$

Consider now the terms in the denominator of the right-hand side of Eq. (10). The quantity A is a function of temperature and, taking the usually accepted values of the parameters involved, we find that A has the values given in Table 1.

TABLE 1. Values of various parameters as functions of temperature

θ (°A)	273	278	283	288	293
A	0·80	1·10	1·48	1·97	2·59
$10^{33} \times B$	1·57	1·87	2·16	2·44	2·69
C	6·13	7·51	8·90	10·30	11·64
$10^7 \times P$	2·35	2·31	2·26	2·23	2·19

Thus $(1 + A)$ is of the order unity and we may neglect that part of the denominator which depends upon drop size provided

$$\left|\frac{P}{a_c}\left(\frac{1}{2x} - \frac{1}{6x^3}\right)\right| \ll 1$$

But P/a_c is generally $\ll 10^{-2}$ and $\left(\frac{1}{2x} - \frac{1}{6x^3}\right)$ is less than 5 provided $x > 0{\cdot}3$. Thus we may neglect that part of the denominator of the right-hand side of Eq. (10) which depends upon x provided $x > 0{\cdot}3$. Substituting the value for a_c from Eq. (2) we are then left with

$$x\frac{dx}{dt} = \frac{B}{m^{3/2}}\left[\frac{y}{3} - \frac{1}{2x} + \frac{1}{6x^3}\right] \quad . \quad . \quad . \quad . \quad (11)$$

where

$$B = \frac{MDk\, p_\theta\, P^{5/2}}{R\theta k + \lambda D\,(dp_\theta/d\theta)}$$

Now as the drop size varies the temperature difference between the drop surface and the ambient air will vary and the coefficient B, which is a function of temperature as shown in Table 1, will also vary. We shall see below however that in many cases of interest the temperature difference between drop and air is so small that no serious error is incurred in selecting a value of B appropriate to the ambient temperature and treating it as a constant. For a specified value of y Eq. (11) can then be integrated graphically to give the time for x to increase from one specified value to another, and values for similar increase in x but with different nucleus massses or different temperatures can be obtained by a simple change in the multiplying factor $B/m^{3/2}$.

Table 2 gives the numerical result of integrating Eq. (11) graphically from 0·6 to x_1 for a selection of values of y and x_1.

TABLE 2. VALUE OF $\displaystyle\int_{0{\cdot}6}^{x_1} \frac{x\,dx}{\frac{y}{3} + \left(\frac{1}{6x^3} - \frac{1}{2x}\right)}$

x_1 \ y	1·0	1·5	2·0	3·0
0·7	0·42	0·18	0·12	0·08
0·8	1·84	0·50	0·30	0·18
0 9	7·23	0·96	0·54	0·30
1·0		1·52	0·82	0·44
1·5		4·86	2·60	1·34
2·0		8·44	4·72	2·51
3·0		16·5	9·94	5·56
4·0		26·2	16·6	9·61

Now suppose, for example, we require to know the time of growth of a droplet from a radius of 2 μ to a radius of 5 μ in an atmosphere at a temperature of 283°A and relative humidity 100·1 per cent, and suppose further that the Na Cl nucleus has a mass of 10^{-14} g. The value of P (from Table 1) is $2{\cdot}26 \times 10^{-7}$ and from Eqs. (2) and (4) we get

$$a_c = 2{\cdot}10 \times 10^{-4}\ \text{cm}, \quad h_c = 3{\cdot}59 \times 10^{-4}$$

We thus require the time of growth from $x = 0{\cdot}95$ to 2·38 with $y = 2{\cdot}79$. From Table 1 the value of B is $2{\cdot}16 \times 10^{-23}$ and the required time is

$$\frac{(10^{-14})^{3/2}}{2{\cdot}16 \times 10^{-23}}(3{\cdot}80 - 0{\cdot}40) = 157\ \text{sec}$$

the quantity (3·80 − 0·40) having been obtained by double interpolation in Table 2.

In passing we may note that when the drop is so large that the effects of the salt content and curvature of the surface are quite negligible, i.e., when the terms in x on the right-hand side of Eq. (11) can be ignored, that equation reduces to

$$a\frac{da}{dt} = \frac{MD\,p_\theta\,h}{R\theta\,(1+A)} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

This is the same as the equation usually quoted for the growth of large drops except for the factor $(1 + A)$ in the denominator of the right-hand side. The presence of this factor arises from making allowance for the difference between the drop-surface temperature and the ambient temperature and the numerical values for A given in Table 1 show that the effect is not negligible for water drops, particularly at the higher air temperatures.

3. Difference between drop temperatures and ambient temperature

From Eqs. (7) and (11) we easily derive

$$\Delta\theta = \frac{\lambda}{kMP}\frac{B}{m^{\frac{1}{2}}}\left[\frac{y}{3} - \frac{1}{2x} + \frac{1}{6x^3}\right] \quad . \quad . \quad . \quad . \quad (13)$$

This equation may be used to evaluate $\Delta\theta$ provided $x >$ about 0·3. When y is large the terms in x may be neglected and we then easily obtain from Eq. (13)

$$\Delta\theta = C\,h \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where

$$C = \frac{\lambda D\,p_\theta}{R\theta k + \lambda D\,(dp_\theta/d\theta)}$$

Values of C are given in Table 1 and we see that with large supersaturations the drop temperature exceeds the ambient temperature by an amount of the order $10\,h$. Thus even in a relative humidity of 105 per cent $\Delta\theta$ is only about 0·5°C and in view of the slow variation of B with θ we are thus justified in treating B as a constant when integrating Eq. (11).

Acknowledgment

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551.501.45 : 519.2

On the correlation of vectors

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Summary

The regression equation for a random vector depending on two others is developed in tensor notation by known methods and total and partial correlation numbers are defined.

1. Introduction

There are many problems in which a meteorologist must consider the statistical correlation between two or more random vectors. For example, it seems likely that simple regression equations could be used as a very rapid means of making certain types of short-period forecasts of upper winds. The aim of this paper is to show how appropriate formulae may be developed. Two examples, the regression equation for a vector depending on one other, and that for a vector depending on two others, are considered in detail. The methods employed are well known and the results for the first example have been obtained by Masuamya (1939) with a technique which is equivalent to that used here; however, the results for the second example do not seem to have been published previously, and the definition of the partial correlation numbers is believed to be new.

Just as the interdependence of two scalar random variables v and ψ may be studied with the aid of their covariance $\overline{v'\psi'}$ so may the interdependence of two vector random variables $\mathbf{u}$ and $\mathbf{\Psi}$ with the aid of their covariance tensor $\overline{v_i'\psi_j'}$. Each of the components of this tensor is equal to the covariance of one of the components of $\mathbf{u}$ with one of the components of $\mathbf{\Psi}$. It should be noted that it is not necessarily symmetric. In the following sections it is shown how this tensor may be used in regression problems. The notation which will be used is summarized in Table 1.

TABLE 1. Notation

Random variable	Deviation from mean	Variance	Reciprocal variance
$\mathbf{u}$	$\mathbf{u} - \overline{\mathbf{u}} = \mathbf{u}'$	$\overline{v_i' v_j'} = \phi_{ij}$	Φ_{ij}
$\mathbf{\Psi}$	$\mathbf{\Psi} - \overline{\mathbf{\Psi}} = \mathbf{\Psi}'$	$\overline{\psi_i' \psi_j'} = \gamma_{ij}$	Γ_{ij}
$\boldsymbol{\omega}$	$\boldsymbol{\omega} - \overline{\boldsymbol{\omega}} = \boldsymbol{\omega}'$	$\overline{\omega_i' \omega_j'} = \chi_{ij}$	X_{ij}
Covariances	$\overline{v_i' \psi_j'} = \rho_{ij}$	$\overline{v_i' \omega_j'} = \tau_{ij}$	$\overline{\psi_i' \omega_j'} = \beta_{ij}$

2. The regression of one vector on one other

The problem is to estimate $\mathbf{u}'$ given $\mathbf{\Psi}$ and assuming that the regression is linear. This estimate will be $\Xi_{ij}\psi_j'$ where the tensor Ξ_{ij} is chosen so that the estimate is the best possible. What is meant by ' best ' is a matter of definition. The simplest definition, which will be used in this paper, is that the mean square error in the estimate of $\mathbf{u}'$, $\overline{(v_i' - \Xi_{ij}\psi_j')^2}$, should be a minimum*. This requires that its derivative** with respect to the tensor Ξ_{ij} should be zero. That is,

$$\frac{\partial}{\partial \Xi} \overline{(v_i' - \Xi_{ij}\psi_j')^2} = 0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Hence

$$\rho_{ik} - \Xi_{ij}\gamma_{jk} = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

so that

$$\Xi_{ip} = \rho_{ik}\Gamma_{kp} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

With this expression for Ξ_{ip}, the equation for the mean square error in the estimate of $\mathbf{u}$ is

$$\begin{aligned}\overline{(v_i' - \Xi_{ij}\psi_j')^2} &= \phi_{ii} - 2\rho_{ij}\rho_{ik}\Gamma_{kj} + \gamma_{jm}\rho_{ik}\Gamma_{kj}\rho_{is}\Gamma_{sm} \\ &= \phi_{ii} - \rho_{ik}\Xi_{ik} \quad . \quad . \quad . \quad . \quad . \quad (4)\end{aligned}$$

* It can easily be shown that the slightly more general definition, that $\overline{(v_i' - \Xi_{ij}\psi_j')(v_k' - \Xi_{km}\psi_m')}A_{ik}$ should be a minimum, where A_{ik} is any constant tensor, leads to the same expressions for the regression coefficients but modifies those for the correlation numbers.

** The derivative of a function of a tensor $f(a_{ij})$ is defined as the limit of the coefficient of $(\delta a)_{ij}$ in $\{f[a_{ij} + (\delta a)_{ij}] - f(a_{ij})\}$ as $(\delta a)_{ij}$ tends to zero in any manner. It can easily be seen that the rules for finding such derivatives are similar to those for ordinary scalar derivatives, though in evaluating them care must be taken to remember that the labelling of dummy suffices is arbitrary.

3. The correlation ratio

It is not possible to find any tensor which bears the same relation to the covariance tensor as the scalar correlation coefficient does to the scalar covariance. It is, however, very desirable to find some number which indicates simply and directly the degree of dependence of one vector on another. Eq. (4) shows that $\rho_{ik}\,\Xi_{ik}$ is the amount by which the mean square error of the best estimate of $\mathfrak{v}$ using knowledge of Ψ is less than the mean square fluctuation of $\mathfrak{v}$ itself. Thus the number $[\nu(\mathfrak{v}\,;\Psi)]^2 = \rho_{ik}\,\Xi_{ik}/\phi_{ss}$ must be positive or zero and cannot exceed unity. When it is zero, knowledge of Ψ does not affect the estimate of $\mathfrak{v}$ and when it is unity Ψ completely determines $\mathfrak{v}$. Thus ν^2 has the most useful properties of a correlation coefficient. It happens that in the one-dimensional case ν (not ν^2) degenerates into the ordinary correlation coefficient, so $\nu(\mathfrak{v}\,;\Psi) = [\rho_{ik}\,\Xi_{ik}/\phi_{ss}]^{\frac{1}{2}}$ will be called 'the correlation ratio of $\mathfrak{v}$ on Ψ.' It should be noted that $\nu(\mathfrak{v}\,;\Psi)$ is not necessarily equal to $\nu(\Psi\,;\mathfrak{v})$.

4. The regression of one vector on two others

The problem is to find the best estimate of $\mathfrak{v}'$ given both Ψ and $\boldsymbol{\omega}$, and assuming that the regression is linear. The treatment is similar to that of Section 2. The best estimate will be $H_{ij}\,\psi_j' + Z_{ij}\,\omega_j'$ where H_{ij} and Z_{ij} are chosen so that $\overline{(v_i' - H_{ij}\,\psi_j' - Z_{ij}\,\omega_j')^2}$ is a minimum. This requires that its derivatives with respect to both H_{ij} and Z_{ij} vanish. Hence

$$\overline{(v_i' - H_{ij}\,\psi_j' - Z_{ij}\,\omega_j')\,\psi_k'} = 0 \quad . \quad . \quad . \quad (5)$$

and

$$\overline{(v_i' - H_{ij}\,\psi_j' - Z_{ij}\,\omega_j')\,\omega_k'} = 0 \quad . \quad . \quad . \quad (6)$$

i.e.,

$$\rho_{ik} - H_{ij}\,\gamma_{jk} - Z_{ij}\,\beta_{kj} = 0 \quad . \quad . \quad . \quad . \quad (7)$$

and

$$\tau_{ik} - H_{ij}\,\beta_{jk} - Z_{ij}\,\varkappa_{kj} = 0 \quad . \quad . \quad . \quad . \quad (8)$$

Multiply (7) by Γ_{km} and (8) by B_{km} and subtract to obtain the equation,

$$-Z_{ij}\,(\beta_{kj}\,\Gamma_{km} - \varkappa_{jk}\,B_{km}) = \tau_{ik}\,B_{km} - \rho_{ik}\,\Gamma_{km} \quad . \quad . \quad . \quad (9)$$

Similarly

$$-H_{ij}\,(\beta_{jk}\,X_{km} - \gamma_{kj}\,B_{mk}) = \rho_{ik}\,B_{mk} - \tau_{ik}\,X_{km} \quad . \quad . \quad . \quad (10)$$

These equations can readily be solved for Z_{ij} and H_{ij}, with the result,

$$Z_{ip} = \rho_{in}\,\Theta_{pn} - \tau_{ik}\,B_{km}\,\gamma_{mn}\,\Theta_{pn} \quad . \quad . \quad . \quad (11)$$

$$H_{ip} = \tau_{in}\,\Theta_{np} - \rho_{ik}\,B_{mk}\,\varkappa_{mn}\,\Theta_{np} \quad . \quad . \quad . \quad (12)$$

where Θ_{jn} has been written for the reciprocal of $\beta_{jn} - \varkappa_{kn}\,B_{km}\,\gamma_{mj}$.

With these values for Z_{ij} and H_{ij} the equation for the mean square error of the estimate of $\mathfrak{v}$ can easily be shown to be

$$\overline{(v_i' - H_{ij}\,\psi_j' - Z_{ij}\,\omega_j')^2} = \phi_{ii} - H_{ij}\,\rho_{ij} - Z_{ij}\,\tau_{ij} \quad . \quad . \quad . \quad (13)$$

5. The partial and total correlation numbers

It does not seem possible to define any numbers describing the partial correlation of vectors which are strongly analogous to scalar partial correlation coefficients. However, the numbers $[\mu(\mathfrak{v}\,;\Psi \mid \boldsymbol{\omega})]^2 = H_{ij}\,\rho_{ij}/\phi_{ss}$ and $[\mu(\mathfrak{v}\,;\boldsymbol{\omega} \mid \Psi)]^2 = Z_{ij}\,\tau_{ij}/\phi_{ss}$ are both positive or zero and their sum cannot exceed unity. They indicate respectively the dependence of $\mathfrak{v}$ on Ψ when $\boldsymbol{\omega}$ is kept constant, and of $\mathfrak{v}$ on $\boldsymbol{\omega}$ when Ψ is kept constant. Thus $\mu(\mathfrak{v}\,;\Psi \mid \boldsymbol{\omega})$ will be called 'the partial correlation number of $\mathfrak{v}$ on Ψ when $\boldsymbol{\omega}$ is constant.' The square root is taken to preserve the analogy with Section 3.

The sum $[\mu(\mathfrak{v}\,;\boldsymbol{\omega} \mid \Psi)]^2 + [\mu(\mathfrak{v}\,;\Psi \mid \boldsymbol{\omega})]^2$ indicates the total improvement in our knowledge of $\mathfrak{v}$ on being given both Ψ and $\boldsymbol{\omega}$. The square root of this sum will be called 'the total correlation of $\mathfrak{v}$ on Ψ and $\boldsymbol{\omega}$.'

A particular result which is often useful is that if there is no partial correlation of u on Ψ, then

$$\rho_{ik} B_{mk} x_{mn} - \tau_{in} = 0 \quad . \quad . \quad . \quad . \quad (14)$$

so,

$$\rho_{ik} = \tau_{in} X_{nq} \beta_{kq} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

6. Application and extensions

The formulae found in this paper may look formidable to those not familiar with tensors, but in practice their use would not involve much labour. The computation of the variance and covariance tensors, which is known to be quite practicable in the case of wind measurements, would seem to be a much larger task than the subsequent evaluation of the regression tensors and correlation numbers.

The methods used in this paper can be applied to a greater number of random variables and also to the case of non-linear regression. Non-linear regression is, however, rather complicated and not entirely straightforward.

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551.510.42

Natural removal of atmospheric pollution during fog

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(Manuscript received 21 November 1953, in revised form 4 December 1953)

Summary

The average lives of smoke particles and sulphur-dioxide molecules in the air are compared in average and foggy conditions. Possible mechanisms of removal during fog are considered.

1. Average conditions

In an earlier paper (1950) I have shown that questions concerning the yearly average concentrations of pollution in Britain, and their relation to the mass of pollution emitted, are matters of simple algebra. They require some or all of the following information :

m = the yearly amount of each form of pollution emitted over Great Britain . (1)

l = the yearly amount blown to sea . . . . . (2)

$m - l$ = the yearly amount deposited on this island . . . . . (3)

c = the average concentration of each form of pollution in surface air . . (4)

A = the area of Britain concerned (excluding the outer halves of Scotland, Wales and the Cornish peninsula it is 65,900 mi²) . . . . . (5)

cAh = the average total pollution over Britain . . . . . (6)

u = the average wind velocity (9 mi/hr) . . . . . (7)

T = the time needed for polluted air to reach the coast, averaged over the whole of Britain, for a whole year . . . . . (8)

t = the average life of each form of pollution before deposition on land, assuming the land to be of infinite extent . . . . . (9)

There are two simple equations connecting these quantities, namely

$$lT = cAh \qquad \text{and} \qquad m/l = \exp(T/t)$$

Enough information was available about all but two of the quantities for rough estimates to be made, by means of the above equations, of the remaining two (h and t).

I have found a way to check my former estimate of T, and think it was too high by at least 20 per cent. The purpose of this note is to draw attention to the revised estimates of t and h; and to attempt an evaluation of t during the London fog of 5-9 December 1952.

The earlier estimate of T was founded on the fact that the area A would make a circle of 145-mi radius, whence a reasonable value of T would appear to be $145/9 = 16$ hr. If Britain is divided into four equal areas (S. Scotland, N. England, SE. England, SW. England and E. Wales) and the distances to the sea from each area are averaged, weighted according to the directional wind frequencies, the average distance downwind to the sea is found to be 112 mi. The new estimate of T is therefore $112/9 = 12\frac{1}{2}$ hr.

This can be checked by the following independent method, using the maps published in my earlier paper. The amount of pollution blown to sea (l) can be calculated in terms of the concentration at the coast (s), the length of coast line (C), the scale height (h), and the average wind velocity (u). If it can be assumed that on-shore winds are quite free from pollution, the observed average concentration of pollution is about half the average concentration in off-shore winds and $l = sChu$, approximately.* The values of h and u in this expression will be similar to the values defined by (6) and (7) above. Taking the distance from headland to headland round the area marked (on the 1950 maps), $C = 1{,}350$ mi. From the same maps, the average concentration of smoke $s = 3{\cdot}1 \times 10^{-11}$ g/cm^3 making $l/h = 2{\cdot}80$ g cm^{-1} sec^{-1}. This may be compared with $l/h = 1{\cdot}69$ from the original estimate of T, and $l/h = 2{\cdot}26$ from the revised estimate of T.

In the circumstances, the revised estimate seems acceptable, and we find the values given in Table 1. The quantities in brackets have been obtained indirectly. The quantity with a dagger is the most questionable of the direct observations.

TABLE 1

$T = 12\frac{1}{2}$ hr $\qquad h = (212)$ m

	million tons/year	
	Smoke	SO_2
mass emitted (m)	2·3	5·0
mass deposited (m-l)	0·8†	(3·9)
mass blown to sea (l)	(1·5)	(1·1)
average life over land (t)	(28 hr)	(8¾ hr)

2. The London fog, 5-9 December 1952

During a prolonged period of calm weather, the only means of removing pollution is by deposition. It then often happens that there is a height above which the air increases sharply in temperature and purity, and below which the air can mix, though usually rather slowly, right down to ground level. There can be little or no transfer of air across the inversion level. If its height is h' and the concentration of pollution at all levels beneath it is c', the total mass of pollution over an area A' of ground is $c'A'h'$.

It is possible for such inversions, with little or no surface wind, to persist for several days at a time; this may occur if a fog forms which is dense enough to reflect solar radiation and prevent its energy from being absorbed below the inversion level. We know only too well that the concentrations of atmospheric pollution can be dangerously high when large town or industrial districts are in fog for several days at a time. The London fog of 5-9 December 1952 was the first of its magnitude in which numerous observations of atmospheric pollution were made. This allows a possibility of discovering, in part, what became of the pollution emitted into it.

* If the multiplier $\cos\theta$ were introduced to allow for winds blowing obliquely off-shore, without partly compensating by allowing for variations of pollution with wind direction, this expression would have to be multiplied by a factor of approximately $2/\pi$.

Consider a rectangular area including Edgware and Chingford in the north, Hornchurch and Dartford in the east, Croydon and Surbiton in the south, Hounslow and Pinner in the west. This area of 450 mi^2 lies mainly in the bottom of the London Basin and must have a population of about 8 million, nearly 1/6 of that of the United Kingdom. It will emit less than 1/6, say 1/10 of the total pollution emitted in Britain; i.e., it will emit 0·2 million tons of smoke and 0·5 million tons of SO_2 per year. In winter it probably emits daily 700 tons of smoke and 1,400 tons of SO_2. I shall take these as the rates of emission during the great fog.

The top of the fog was about 500 ft above ground, and the mass of air below this height, 450 mi^2 in extent, is 226 million tons. One day's pollution would thus produce concentrations of 700/226 = 3·1 parts per million of smoke and 6·2 parts per million (by weight) of SO_2; in more familiar units, 4·0 mg/m^3 of smoke, and 8·0 mg/m^3 of SO_2.

The observed concentrations of smoke and SO_2 did not nearly reach these values in the centre of London at ground level, and they are unlikely to have done so elsewhere. The average concentrations at six places in London during the five days of fog were 2·2 mg/m^3 of smoke and 2·1 mg/m^3 of SO_2. Thus the average life of a smoke particle must have been 2·2/4·0 of a day = 13 hr; and the average life of a SO_2 molecule was 2·1/8·0 of a day = 6 hr.

3. Discussion

These life-times are interesting in several ways. First, they should be compared with the time needed by air to pass right across London; air drifting at ½ mi/hr would take 40 hr to go 20 mi. The life-times are much shorter than this, suggesting that the pollution was removed from the air of London by deposition and not by ventilation. London housewives will remember vividly what good reason they had to support this contention.

Second, the life-time of pollution in fog is apparently shorter than in average weather. This unexpected result becomes acceptable when we recognize that the life-time before *deposition* is all we are concerned with, and that pollution may rise in normal weather to 40,000 ft whereas in the fog it could not rise above 500 ft. Thus although the air turns over much more slowly in fogs it has not so far to turn as in normal weather.

Third, there is need to consider whether the fog droplets themselves help in the deposition of pollution. The standard instruments for measuring pollution do not collect particles that are falling faster than about 0·3 cm/sec (Leicester Report 1945), so they do not collect any but the very smallest fog droplets, less than 10^{-3} cm in diameter. The pollution caught by practically any fog droplet is therefore lost to the sampling instrument even before the droplet sinks to the ground. There is little difference between the chances of pollution sticking to fog droplets and to the ground or other outdoor objects, all of which are wet; what matters is the collision frequency of pollution with unit area of surface, multiplied by the total area available.

With a liquid-water content of 1 g/m^3 in a typical fog (Bilham 1938), if the droplets are all 2×10^{-3} cm in diameter, the total water surface in the 152 m^3 of air above 1 m^2 of ground works out as 46 m^2. This must be compared with the surface area of solid objects (grass, trees, buildings etc.) per square metre of London, and allowance must be made for differences in collision frequency since this is likely to differ between droplets and stationary objects and between large smooth surfaces and small irregular surfaces in the free air. All in all, one would guess that at least as many particles of smoke or molecules of sulphur dioxide were collected by droplets in the great fog as by solid objects coated with moisture.

Having embarked on the question of collisions, we must also consider collisions between two particles of pollution. The size distribution of smoke particles on a hazy day in Teddington has been measured (Meetham 1950); half the surface area was provided by particles less than $3·5 \times 10^{-5}$ cm in diameter (assuming the particles to be nearly spherical, as indeed the smaller ones were). If their density is that of graphite, 2·3 $g\ cm^{-3}$, and if their concentration in the air to a height of 152 m was 2·2 $mg\ m^{-3}$, the total surface area of smoke was 2·5 m^2 over each square metre of ground, assuming the smoke particles were not wetted. This is not negligible either for absorption of sulphur dioxide, or for combination with other smoke particles.

To take the second effect first, there is some observational evidence that smoke particles do combine in the open air to form chains ' like floating cobwebs ' visible to the naked eye (Donora 1949); the chain-forming propensities of dense artificial carbon smokes are fairly well known.

The other effect, the absorption of sulphur dioxide by smoke particles, is likely to be highly important. The collisions of SO_2 molecules with smoke particles were probably about 2·5/46, i.e., about 1/20th as numerous as collisions with fog droplets. But it is quite conceivable that

SO_2 finds it 10 or 100 times as easy to stick to a smoke particle as to a fog droplet, in which case more SO_2 could have been captured by smoke particles than by either fog droplets or solid surfaces. The standard volumetric-measuring instrument does not measure SO_2 attached to smoke, because the smoke is filtered out before the air passes through the hydrogen-peroxide solution which collects the SO_2 (the amount of gaseous SO_2 stopped by the smoky filter has been shown to be small).

4. Conclusions

When all the above considerations are taken into account, it does not seem unreasonable that both smoke and SO_2 can have a shorter life-time in fogs than in normal weather. If so, we should be thankful that when ventilation fails the secondary scavenging processes which come into operation are so powerful. *Furthermore, measures taken during a fog to reduce the emission of pollution are likely to produce a noticeable reduction in air pollution within a few hours.*

One important point must still be made clear. The standard instrument fails to measure sulphur dioxide attached to smoke, and fails to collect particles falling faster than about 0·3 cm/sec. Unfortunately the human lung partly succeeds where the instrument fails. It collects some of the fog droplets, and some of the smoke particles whether they have attached SO_2 or not; also some of the free SO_2 molecules. Much of the pollution is, however, breathed out again without having been caught by the interior surfaces of the lung. It is still not known whether SO_2 is readily oxidized in fog droplets, on smoke particles, or when caught by the lung; but oxidation to sulphuric acid may ultimately turn out to be the explanation why relatively low concentrations of pollution can endanger human life.

Little progress has been made as yet with methods of measuring atmospheric pollution in other forms than smoke and sulphur dioxide. Measurements of carbon monoxide and other gases are undoubtedly needed, but it may prove still more important to measure the SO_2 and SO_3 attached to smoke and dissolved in fog droplets.

Acknowledgment

I am indebted to the Director of the National Physical Laboratory for permission to publish this paper.

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551.574.1

Drop sizes in sea mists

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1. Introduction

Though considerable data on the drop sizes in rain have been collated by Best (1950) there seem to be few data for sea mists. A feature of the climate of the NE. coast of England is the 'haar,' probably a type of advection fog, which occurs in the late spring in the forenoon and evening (Manley 1935). As it rarely persists for long periods it was thought that it might afford drop-size data in a range of size smaller than those quoted by Best.

2. Apparatus

Beside drop-size distribution, mass content and visual range were measured, together with the usual meteorological data.

Sampling for drop size was done by exposing microscope slides coated with magnesium oxide, as recommended by May (1950), on the roof of the Dove Marine Laboratory, Cullercoats. The slides were sized, using a microscope equipped with Kodak graticule.

For mass content, the atmospheric air was aspirated by a fan through screens of silk cloth mounted on embroidery rings in an open-ended metal cylinder, similar to the device used by Houghton and Radford (1938). The screens were weighed dry and after 10-min exposure and the quantity of air aspirated in this time calculated from the speed of suction.

Visual range was estimated from observation of a set of objects in Cullercoats Bay, located on a map.

Typical results are given in Table 1. A series of summation curves to illustrate the ageing of a fog – increasing mean size – are shown in Fig. 1.

TABLE 1.

Case number	Time (GMT)	Dry-bulb temperature (°C)	Wet-bulb temperature (°C)	Relative humidity (%)	Wind velocity (ft/sec)	Predominant drop diam. (μ)	Visual range (yds)	Mass content (g/m^3)	Remarks
1	12	—	—	—	—	16	—	—	
2	16	9·0	8·6	95	8·1	32	400	—	dense
3	17	8·4	8·0	95	9·0	16	360	—	very dense
4	18	8·8	8·0	90	13·6	16	420	—	started to clear
5	11	8·0	7·0	87	10·6	16	390	—	
6	1130	7·6	7·0	92	4·2	16	420	—	
7	12	7·5	7·0	93	5·6	16	3000	—	clearing up
8	1220	7·8	7·1	92	5·5	16	3200	—	clearing up
9	1630	8·3	8·0	96	calm	16	525	—	
10	1750	8·0	7·8	97	calm	32	420	—	
11	2030	7·2	7·1	98	calm	45 to 80 (fog ageing)	420	—	about to rain
12	0545	17·7	16·0	84	10·5	11	3240	—	clearing up
13	0610	17·5	16·0	86	13·3	11	3240	—	clearing up
14	1630	8·5	7·0	81	11·1	20	3200	0·005	
15	09	3·0	2·7	96	1·1	16	280	0·0042	very dense
16	0930	2·2	1·7	92	1·1	22	250		
17	09	1·5	1·0	92	2·92	32	420	0·0083	dense
18	12	11·8	10·9	90	3·0	37	100	0·0135	very dense
19	1230	10·6	10·2	95	3·8	34	120	0·095	very dense
20	10	10·6	10·2	95	0·9	22	100	0·0131	very dense
21	1930	11·4	11·0	95	1·2	32	150	0·0123	very dense
22	2020	10·8	10·6	98	1·1	45	100	0·023	
23	2130	11·0	10·0	89	1·3	42	280	0·0115	dense

3. Discussion

It will be noted that the mean drop diameter does not vary considerably in fogs in spite of variation in the meteorological factors concerned. It will be seen that :

(a) Fog can develop in unsaturated air where relative humidity is as low as 85 per cent (cases 12 and 13).

(b) Strong wind is one of the most important factors of fog dissipation (cases 4 and 13), probably because of associated turbulence.

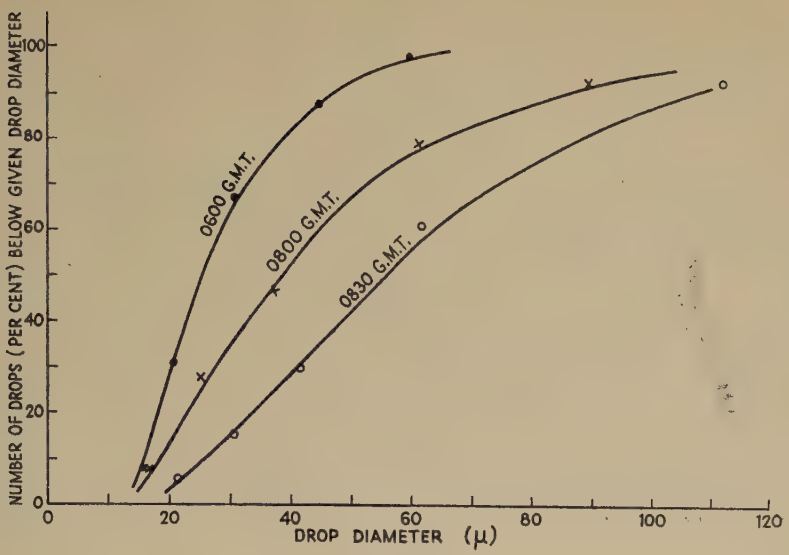

Figure 1.

(c) According to Trabert's law (1901), the visual range should be given by the ratio of mean drop size to mass content. Merrington and Richardson (1950) using dense smokes and indoor measurements of visual range have qualitatively confirmed this law. A comparison of cases 15 and 17 or 18 and 23 of the present results does not confirm this law, though caution must be exercised in deriving conclusions from the difficult process of estimating visual range out of doors. Recently it has been shown (aufm Kampe and Weickmann 1952, Best 1953) that Trabert's law is only true when fogs of the same drop-size distribution are compared.

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551.521.11

Sunshine in the southern Sudan

By J. Pepper

In the course of an investigation by the Meteorological Office reference was made to the note on ' Sunshine in the Southern Sudan ' by W. D. Flower (*Q.J.*, **65**, p. 448, 1939) :

When Eq. (1) (p. 449),

$$S = (100 - C)(1 + tC - a'W)$$

was checked, it was found that the values in Table 2 (p. 450) of mean cloud amount were in error. Not all the values in this table have been worked out, but I append the values I have found for the mean for Meridi (4° 55′N, 29° 30′E) (first row in the table) :

Mean daily amount of cloud in tenths

J	F	M	A	M	J	J	A	S	O	N	D
6·7	6·7	6·3	6·1	6·4	6·9	7·5	7·1	5·5	5·8	6·8	7·8
(4·0)	(4·7)	(3·8)	(3·5)	(3·8)	(4·1)	(4·6)	(4·2)	(3·2)	(3·4)	(4·0)	(4·8)

The values in brackets are those of the author.

Meteorological Office,
Harrow.
25 July 1953.

551.558.1 : 551.576.11

The stable sub-cloud layer and convective updraughts

By A. F. Bunker

It is desired to correct a misquotation and omission of pertinent data that occur both in the review paper by Ludlam and Scorer (*Q.J.*, **79**, p. 336, 1953) and in the paper by Ludlam (*Q.J.*, **79**, p. 430, 1953). In reviewing the paper by Bunker, Haurwitz, Malkus and Stommel (*Pap. Phys. Ocean. Met.*, M.I.T. and Woods Hole Ocean. Inst., **11**, No. 1, 1949) describing measurements of temperature and humidity over the Caribbean Sea, Ludlam and Scorer have stated that we interpret the existence of a stable sub-cloud layer as evidence that updraughts in the clouds do not extend down into the adiabatic layer. They then try to show that this reasoning is incorrect. Actually, the reasoning presented was not based on the existence of a stable layer but on the records of a water-column accelerometer carried in the sounding aircraft. This instrument records the accelerations normal to the plane of flight imparted to the aircraft by the turbulent air. A paragraph may well be repeated here since distribution of the paper in the British Isles is rather limited (it had been pointed out in the preceding paragraph that the average mean deviations in the mixing ratio were essentially the same under the cumulus as in the clear areas) :

> Substantiating evidence that convection currents are not maintained far into the homogeneous layer is given by accelerometer records of the vertical motions of the plane as it flew under clouds and in clear areas. Table 32 gives the percentages of time that the accelerations of the plane were in the ranges 0·0 to 0·04 g; 0·04 g to 0·08 g, and greater than 0·08 g, for both clear and cloudy conditions. The bottom row gives the averages for each range and cloud condition above the plane. The small difference shown in the turbulence in the homogeneous layer under the clouds and in clear areas indicates that convection in the cloud layer does not produce any recognizable increase of the turbulence in the layer below it.

TABLE 32. ACCELERATIONS IMPARTED TO AIRPLANE

Date	Altitude (m)	Clear conditions			Under clouds		
		0·0-0·04 g	0·04-0·08 g	0·08 g	0·0-0·04 g	0·04-0·08 g	0·08 g
14 April	300	81	17	2	85	15	0
23 April	300	84	13	3	90	10	0
14 April	150	81	17	2	76	22	2
13 April	135	78	19	3	73	23	4
26 April	150	82	16	2	77	20	3
Average percentages		81·2	16·4	2·4	80·2	18·0	1·8

Another observation presented in the paper that should be considered is the existence of a minimum of turbulence in the stable layer near the cloud base. This minimum is interpreted as the result of the damping out of both the mechanical turbulence of the rough air below and the convective motions in the cloud layer above the stable layer. This region of minimum turbulence, in conjunction with little difference between the deviations of the mixing ratios under clouds and in the clear air and the small difference in the degree of turbulence, is the basis for the authors' statement that :

> Over the ocean the convection currents producing the clouds neither originate in nor extend an appreciable distance down into the homogeneous layer . . . . . The convective activity which eventually leads to cloud formation probably starts at the top of the homogeneous layer, rather than deep in the layer where the air is mixed more rapidly.

These observations constitute the evidence on which the foregoing statement was based. It is possible that other interpretations of the observations can be made, but criticism of the conclusion must take into account the observations and under no circumstances should the observations be ignored or their existence unacknowledged.

The only qualifications that might be made to the statement concerning the continuity of convective motions between the adiabatic layer and the cloud layer is that the process is likely to operate in atmospheres such as the trade winds, which are slightly stable and subsiding and are underlain by a surface which has temperature gradients of the order of 1°C/500 mi. It has been proved in innumerable cases that over heated land and in regions where the ocean surface is much warmer than the air, there is continuity between the convective motions of the air parcels in the turbulent region and the motions of the cumulus clouds. However, one must not reason, as Ludlam and Scorer appear to do, that because the stable layer is common to both continental and maritime air masses and is frequently associated with the thermals and cumulus clouds, continuity of motion must necessarily exist between clouds and the adiabatic layer in a subsiding air mass passing over a water surface of the same temperature as the air.

They are undoubtedly right in suggesting that convective activity, in addition to general subsidence, creates a stable layer beneath the cloud bases. Such a stable layer would appear near the lifting condensation level in contrast to the height of a turbulence inversion which depends upon the turbulent energy of the air of the mixed layer and the potential temperature variation with height. If such a stable layer is created beneath a turbulence inversion it may be strong enough to resist the dissipative effects of turbulence and an inversion will exist at the new lower level that is a combination of convection, subsidence, and turbulence effects. Such appears to be the case in the North Atlantic trade-wind region.

However, it is not a necessary condition for the formation of a stable layer that thermals originate at or near the ground or water surface. A slow large-scale circulation of the order of 20 km would be more than sufficient for its formation. The existence of such a circulation or convergence is borne out by the average heights of the bases of the stable layer which is 607 m under clouds and 554 m in the clear air. In fact it is not even necessary to consider a circulation that includes the lowest turbulent layer, for Stommel showed in the same paper that an excellent fit of the humidity curve is obtained if the subsidence is set equal to zero at the base of the stable layer. Thus it would appear that the stable layer will form regardless of the height of origin of the convecting air. It is of interest to note that in the overland air mass studied by Ludlam (*Q.J.*, p. 431, 1953) on 16 April 1952, which is most nearly similar to the trade-wind air masses as regards subsidence, lapse rate, height of turbulence inversion, and intensity of inversion, he was unable to find any observational evidence of continuity between the motions and temperatures

of the clouds and the thermals in the adiabatic layer. If he had used a fast-response thermometer, as the observers on the Caribbean field trip did, instead of a slow-response one as he seems to imply that he did, he might have detected thermal differences under the cloud base, thereby proving or disproving continuity. His justification for believing that continuity did exist in this borderline case was that its existence could hardly be doubted. On the basis of his experience with land and polar sea cumulus it is natural that the he should assume continuity even if he failed to observe it. However, observational proof would be desired by one accustomed to observing both land cumulus and cumulus in a subsiding atmosphere moving over a uniform ocean.

Woods Hole Oceanographic Institution,
Woods Hole,
Mass., U.S.A.
23 September 1953.

Reply

By F. H. Ludlam and R. S. Scorer

We are not convinced, even after reading Bunker's welcome rejoinder, by evidence suggesting the absence of thermals in a shallow layer immediately below trade-wind cumulus. In our review we did not attempt to show that such an interpretation of the observations is wrong, but only that the shallow stable layer is probably also often a feature of convection in which thermals do rise from near the surface into the clouds. The stable layer was found, not only at Lasham (where Bunker rightly says that the association between clouds and thermals was assumed), but also at Cranfield, where gliders were readily able to use thermals to enter the clouds (Ludlam 1953). We suppose that the temperature-excess and upward speed of thermals (and therefore the ' turbulence ' encountered by a powered aircraft) here also decrease in the stable layer.

Imperial College,
London.
29 October 1953.

551.577.2 (62) : 551.577.3 : 551.515.8

Rainfall over Egypt

By K. H. Soliman

May I draw the attention of your readers to the need for amplifying the references in my paper which recently appeared in the *Quarterly Journal* (**79**, p. 389, 1953) ? The following references should be added to complete the bibliography on the subject :

Ali, F. M.	1952	' Outstanding variations in rainfall on the north coast of Egypt,' *Bull. Inst. Fouad I du Désert*, **2**, No. 1.
Bleeker, W. and Andre, M. J.	1951	' On the diurnal variation of precipitation, particularly over central U.S.A.,' *Quart. J. R. Met. Soc.*, **77**, p. 260.
El-Fandy, M. G.	1946	' Barometric lows of Cyprus,' *Ibid.*, **72**, p. 291.
	1948	' The effect of the Sudan monsoon low on the development of thundery conditions in Egypt, Palestine and Syria,' *Ibid.*, **74**, p. 31.
	1950	' Troughs in the upper westerlies and cyclonic developments in the Nile Valley,' *Ibid.*, **76**, p. 166.

Meteorological Department,
Koubry-El-Quobbah P.O.,
Cairo.
29 September 1953.

551.576.11 : 551.578.1

Showers from cumulus congestus clouds

By L. Alpert

In his paper ' Observation of rain from non-freezing clouds ' (*Q.J.*, **77**, p. 33, 1951) E. J. Smith points out the paucity of published data on instances of appreciable rain falling from non-freezing clouds. To add to the literature on the subject, I should like to point out that references were made to this phenomenon in my paper ' Weather over the tropical eastern Pacific Ocean, 7 and 8 March 1943 ' (*Bull. Amer. Met. Soc.*, **27**, p. 384, 1946).

Showers falling from cumulus congestus clouds were observed on many patrol flights over the tropical eastern Pacific Ocean between Guatemala and the Galapagos Islands during 1941-1943. Owing to the fact that the plane had to be flown on a predetermined course, it was not possible to take soundings or otherwise investigate such rainy cumulus congestus which interested me greatly at the time.

The following is a pertinent passage (p. 386) in the article :

7 March 1943, 13 GCT, 11° 30′N, 89° 22′W

Fc clouds covering 1/10 of the sky prevail, with here and there an isolated cumulus *congestus* (Cg) cloud mass. Photographs of both are taken (Photos. 2 and 3, Plates I and II).

These Cg cloud masses are interesting because they cover a relatively small area and are separated by several miles of clear sky or scattered Fc. This phenomenon we had observed on previous flights, and pilots report it is common. At times, showers are observed to fall from these isolated clouds, although cirrus (ice-crystal) tops are lacking. On a flight 11 December 1942 several such showers, falling from similar clouds, were observed and photographed by the author.

4120 34th Street,
North Arlington,
Virginia, U.S.A.
30 October 1953.

551.509.34 : 551.521.2 : 551.524.36

Night cooling under clear skies

By W. E. Saunders

In my recent paper on night cooling (Saunders 1952) the view was put forward that dew formation was responsible for the evening discontinuity in rates of cooling. This was based on visual observations of dew formation and of temperature readings at the grass level. Since, at the discussion on 18 February 1953, wide differences of opinion were expressed as to the cause of the discontinuity, readers may like to be referred to a paper by J. Seemann and K. Löw (1944). This gives simultaneous observations of temperature at several levels near the ground, and of dew formation as obtained from a dew recorder, and shows that latent-heat release can cause a *rise* of temperature at the 10-cm level and even 200-cm level. In an example reproduced there was a rise of temperature of more than 3°C at the 10-cm level.

I am indebted to Mr. F. Loewe of the University of Melbourne for calling attention to this contribution on the subject.

References

Saunders, W. E. 1952 *Quart J. R. Met. Soc.*, **78**, p. 603.
Seemann, J. and Löw, K. 1944 *Met. Z.*, **61**, p. 158.

Meteorological Office, H.Q. No. 19 Group,
Royal Air Force, Mount Batten, Plymouth.
9 December 1953.

551.511

The conditions for dynamical similarity of motions of a frictionless perfect-gas atmosphere

By G. K. BATCHELOR

(Read 21 October 1953. See *Q.J.*, **79**, p. 224)

PROF. P. A. SHEPPARD : Those atmospheric motions which are smaller in horizontal and vertical scale than the scale height of the atmosphere and which are affected by density gradients or, more precisely, by non-adiabatic lapse rates are important in many contexts, but no great theoretical progress has yet been made with them. This paper, which aims to sort out the important physical parameters for such motions, is therefore very welcome. I have two closely related questions with comments to put to Dr. Batchelor in order to seek a fuller appreciation of the applicability of his results :

Motions which are affected by non-adiabatic lapse rates are in general non-adiabatic in nature; heat transfer is involved (by other than molecular conduction) and it is to such motions that Richardson's number has been applied in the past. Dr. Batchelor however assumes adiabatic motion (Eq. 3) and I wonder what sort of restriction this in fact implies. Does it not mean that the velocity **u** appearing in the equations is either,

(*a*) an ' instantaneous ' value at a ' point,' or

(*b*) the motion is laminar (no eddy fluxes on scales smaller than the motions considered) ?

Excluding (*a*) as unprofitable of discussion, in the present context, we infer from (*b*), if accepted, that the analysis could only apply to an atmosphere without large-scale mean motion and without the mechanical turbulence usually associated with it. Is this in fact the implication of Dr. Batchelor's assumption that his ' equivalent ' atmosphere possesses no horizontal pressure gradient (expressed by the relation $\nabla p_a = \rho_a \mathbf{g}$ on the line preceding Eq. (15)).

If my interpretation is correct, I am left in doubt whether Richardson's number can be applied with theoretical justification to any atmospheric motion except perhaps the initial stage of convection in an otherwise motionless atmosphere. I appreciate the difficulty of setting up a system which is both realistic and amenable to analysis so that my queries and comments are to be regarded as exploratory and not critical.

MR. E. L. DEACON (*partly communicated*) : It appears rather generally to be found in problems of the diffusion of matter from elevated sources, at heights of the order of 100 m, that the Richardson number fails to be of much use in representing the variation of rate of diffusion with the varying meteorological conditions. I would like to know if Dr. Batchelor's analysis could help with this difficulty which I presume to arise, in part, from horizontal non-uniformity (perhaps mainly arising at the boundary) bringing into play the selective buoyancy effects discussed by Priestley and Swinbank (*Proc. Roy. Soc.* A, **189**, p. 543).

As there are great practical difficulties in introducing parameters expressing horizontal non-uniformity, I have wondered recently if this trouble might be mitigated by working with what is perhaps a more fundamental type of Richardson number in terms of the eddy flux of heat (Q) and shearing stress (τ), now that ways of measuring these have been developed. This would be, for a particular level,

$$\frac{Q\,g}{C_p\,T\,\tau\,du/dz}$$

which expresses the rate of working of the buoyancy forces as a fraction of the rate of dynamical working by the shearing stress. This, of course, yields the more customary form of Richardson number if we substitute $K\,d\theta/dz$ for Q and so forth, but this appears to be very undesirable for those layers in which the eddy coefficients for heat and momentum are likely to be far from equal and where the vertical gradient of potential temperature bears little relation to the heat flux.

Obviously the flux form of Richardson number would not provide a solution to all difficulties associated with horizontal non-uniformity but in many cases (large obstacles being absent) the difference from site to site on this score might prove not to be a major factor.

MR. E. GOLD : I should like to ask Dr. Batchelor if the condition that the departures from the adiabatic atmosphere are small, is in fact satisfied in all conditions in the actual atmosphere, e.g., is it satisfied when there is a substantial inversion, or only when there is a lapse rate not far from the adiabatic lapse rate ? Presumably the figure 100 m refers to the scale of the motion and not to the height of the atmosphere over which the motion extends.

DR. G. K. BATCHELOR (*in reply*) : My comment on Professor Sheppard's query is that my assumption that each element of fluid moves adiabatically implies *no* restriction except the exclusion of radiative and molecular transfer of heat. The meaning of the velocity **u** in my equations is his alternative (*a*), viz., the velocity of the fluid at position x at time t, and I cannot see why Professor Sheppard should regard this meaning as unsuitable for the analysis. Perhaps he is worried by the fact that the terms expressing turbulent transfer do not appear explicitly in my equations. The reason why they do not appear is that I have not taken mean values of the equations. Conditions for dynamical similarity are determined by the basic equations of the problem and are the same in form for all kinds of motion, laminar or turbulent, so that there is no need to complicate the analysis by separating the velocity into mean and fluctuating components.

What Mr. Deacon says may be right, and we shall look forward to seeing some measurements which might be used to test his suggestion.

With regard to Mr. Gold's question, one of the conditions for the Richardson number to be the sole determining parameter is that the departure of the temperature from its value for an adiabatic lapse rate should everywhere be small relative to the absolute temperature. Hence if the actual lapse rate is appreciably different from the adiabatic lapse rate, the vertical extent of the motion under consideration must be suitably small; whereas if the actual lapse rate is nearly adiabatic, the vertical extent of the motion can be large. The limitation $L \ll p_a/g\rho_a$ arises for different reasons, and refers, as Mr. Gold presumes, to the characteristic scale of the motion and not to the size of the region which it occupies.

551.513.1

A quantitative analysis of the evolution of large-scale flow with regard to the effect of eddy motion

By F. A. BERSON

(Read 21 October 1953. Presented by Mr. J. S. Sawyer. See *Q.J.*, 79, p. 210)

DR. R. C. SUTCLIFFE : I welcome Dr. Berson's paper as what I believe is the first published attempt to tackle an important problem in dynamical meteorology, the mechanism by which the large-scale mean flow patterns are maintained. We are today familiar with the concept of a general circulation pattern which is at least in part maintained by the smaller-scale turbulence on the ordinary synoptic scale; the persistent mean westerlies and easterlies with the associated mean pressure field, the sub-tropical anticyclones and so on, are only possible because the turbulent motion provides the necessary build-up of momentum against surface drag. It is thus reasonable that the quasi-permanent large-scale anomalies, especially the blocking situations which are so different from the mean general circulation pattern, should be maintained by the fluxes of momentum or vorticity associated with the synoptic turbulence. I was however a little surprised to find from Table 4 that the mean flux of vorticity is greater for the barotropic component than for the eddy component over a period in which the mean long-wave pattern changed rather little.

DR. F. A. BERSON (*in reply, communicated*) : I would have liked to see more discussion and criticism of the paper as it leaves me dissatisfied in various respects. On the other hand, it is gratifying to learn that it seemingly has conveyed to the synoptic meteorologist what I intended it should, viz. the recognition of the fact that the usual distinction between a ' general ' circulation and its perturbations is surely an oversimplification. By implication Dr. Sutcliffe has made this point quite clear in his comments.

In reply to Dr. Sutcliffe's query of the relative magnitude of what he calls the barotropic and eddy components of vorticity flux, I would like to suggest that it is more correct to refer to them as the mean motion and eddy motion components of the (differential) barotropic flux. I agree with him that the values obtained for the former component are on the whole unexpectedly large in comparison to those of the latter. A check on the data (first two columns of Table 4) revealed a standard deviation of the former values considerably in excess of the standard deviation of the latter. This may help to explain, at least in part, the discrepancy. But it ought to be remembered that even with regard to a quasi-stationary three-week average flow at the 500 mb level, there may still exist a considerable unbalance between the local variation $\partial\overline{\zeta}/\partial t$ and the transfer term $-\overline{V}\,.\,\overline{\nabla\eta}$; therefore, since that variation does not differ much from zero, $\overline{V}\,.\,\overline{\nabla\eta}$ will differ from $\overline{V'}\,.\,\overline{\nabla\eta'}$ at any rate.

551.511 : 551.515

On the theory of large-scale disturbances in a two-dimensional baroclinic equivalent of the atmosphere

By P. D. THOMPSON

(Read 21 October 1952; presented by Mr. F. H. Bushby. See *Q.J.*, 79, p. 51)

Dr. R. C. Sutcliffe : Could Mr. Bushby say something about the essential differences between Major Thompson's model and that developed by himself with Mr. Sawyer ?

The use of temperature at one level as a second parameter in the equations is of course formally satisfactory but I should have imagined it to be undesirable in practice because the isotherms at say 500 mb are not likely to give a very good representation of the average horizontal temperature gradient through the atmosphere, especially when we have to deal with sloping frontal zones and rather concentrated isotherms at any one level. The use of thickness through a deep layer would seem more representative.

We must agree that the first results illustrated in the lantern slides are further encouragement to those concerned with numerical predictions.

Mr. M. K. Miles : As can be seen from the maps shown, most of the cyclonic development occurred in the second half of the 24-hr period. Might this not be partly responsible for the discrepancy between the predicted and actual development ?

Dr. A. G. Forsdyke : Referring to the forecast charts which we have been shown, I noticed that although the areas of positive and negative tendencies compared well with the actual as regards position, the magnitudes were, on the whole, too large. A similar tendency to exaggerate the pressure changes has been noticed in some numerical forecasts prepared at Dunstable. I suggest that this may have some connexion with the idea of self-aggravating patterns put forward by Sutcliffe. To give a simple example, Sutcliffe's thermal development theory, used quantitatively, requires cyclonic development forward of a thermal trough. The cyclonic circulation tends to further growth of the thermal trough and so we arrive at the idea of a sort of chain reaction in which the interacting depression and thermal trough grow indefinitely. In practice the forecaster keeps the situation under control, so to speak, by assuming that friction and the climatological control on the movement of the thickness lines through exchange of heat with the undersurface will apply the brake. It is precisely these factors, friction and non-adiabatic heat exchanges, that are neglected in numerical forecasts, and I wonder whether the exaggeration of pressure changes which present methods sometimes give is an indication that the neglected factors ought to be taken account of in some way.

Mr. F. H. Bushby (*in reply*) : In reply to Dr. Sutcliffe, the model adopted by Major Thompson is very similar to the model developed by Mr. Sawyer and myself, the main difference being that we used the thickness of a 500-mb layer as our record parameter instead of the temperature at a particular level. I agree with Dr. Sutcliffe that our parameters are more desirable than those used by Thompson, for the reasons stated by Dr. Sutcliffe.

Replying to Mr. Miles, I would say that when one wishes to test a theory by comparing an instantaneous tendency with an actual change averaged over a 24-hr period, one should choose a situation in which the actual rate of change is nearly constant for the 24-hr period. This was not done in this case, and it is difficult to say whether Mr. Miles's point would account for the apparent discrepancy between actual and computed development.

As regards Dr. Forsdyke's comment, I do not think that ' self-aggravating patterns ' could possibly account for the computed tendency field being larger than the actual tendency field, although this could account for discrepancies between actual and computed changes over a 24-hr period.

Major P. D. Thompson (*in reply, communicated*) : With regard to Dr. Sutcliffe's question and comments, I must certainly agree with Mr. Bushby's reply. Since publication of this paper, I have developed a more general and exact method of deriving the equations for two-parameter baroclinic models, without introducing the notion of a ' level of equivalence.' A consequence of this development is that the optimum choice of variables in a two-parameter system is the mean contour height, integrated with respect to pressure through the entire depth of the atmosphere, and the difference (thickness) between the mean contour height and the height of an isobaric surface next to the ground. This result, of course, is in general accord with the view expressed by Mr. Bushby.

In essence, the question raised by Mr. Miles (and later by Dr. Forsdyke) concerns the propriety of comparing instantaneous tendencies with centred 24-hr changes. Granting that such comparisons are improper and inconclusive, strictly speaking, one should then examine the qualitative connection between tendencies and 24-hour changes, in order to anticipate the nature of discrepancies between them. The 24-hour change may be regarded as the time-averaged value of the instantaneous tendencies over a 24-hour period. In general, therefore, the 24-hour change is less than the maximum tendency in that period. Moreover, if the tendency pattern is not stationary, one might expect that the extreme values of the tendency at a fixed time would be systematically greater than the extreme 24-hour changes centred on that time. According to this argument the patterns of the instantaneous tendency and the centred 24-hour change should still agree. These expectations were realized in the main but beyond good qualitative agreement the comparison is inconclusive.

In replying to Dr. Forsdyke's comment, I should first like to distinguish between the accumulation of effects over a period of time, due to the continued operation of a physical mechanism, and the instantaneous behaviour of the mechanism itself. The latter, of course, is a reflection of the fundamental *physical* process, and the former is its essentially *mathematical* continuation over a period of time. As an example of accumulation, the process of ' self-aggravation ' certainly may account for discrepancies between instantaneous tendencies and centred 24-hour changes. For the same reason, however, ' self-aggravation ' alone cannot account for discrepancies between predicted and observed *finite changes*. That is to say, these discrepancies must be due to defects in the physical mechanism, and not merely to the accumulation of effects over a finite period of time.

Finally, although it is not relevant to Dr. Forsdyke's original question, I feel impelled to take issue with his statements about the mechanism of ' braking,' which acts to retard the development of cyclones. In the first place, it should be noted that the thermal trough not only grows as a result of the increasing cyclonic circulation, but also moves rapidly into coincidence with the pressure trough. According to prevailing theory, the maximum of absolute vorticity will not increase after the pressure and temperature troughs coincide, and it is generally observed that the cyclonic circulation reaches maximum intensity as the axis of the pressure trough approaches the vertical. Owing to the rather non-selective character of frictional action, it is difficult to imagine how the existence of friction will account for the rapid decrease in phase lag between the troughs of pressure and temperature. A more plausible hypothesis, in my opinion, is that the developing cyclone constitutes a gigantic feedback mechanism, in which a partially developed temperature field furthers the growth of cyclonic circulation, and the resulting upward motions in advance of the cyclone force the thermal trough into coincidence with the pressure trough. This mechanism, of course, is crucially dependent on the non-linearity of the system. Such an effect has already been discussed, in fact, in a recent article by Kuo (' The stability properties and structure of disturbances in a baroclinic atmosphere,' *J. Met.*, **10**, p. 235, 1953).

In closing, it is suggested that the very same mechanism which controls the process of ' self-aggravation ' later manifests itself in a process of ' self-containment,' and that cyclones will not grow indefinitely, even in the absence of viscous forces.

551.557

(a) The profile of a jet stream observed 18 January 1952
(b) The profile of a jet stream observed 1 September 1952

By G. W. HURST

(Read 18 November 1953. See *Q.J.*, 78, p. 613 and 79, p. 407)

Dr. R. S. Scorer: Could some information be given about any clear-air turbulence encountered on these flights?

Dr. A. W. Brewer: Mr. Durst has shown by camera-obscura technique that the wind fluctuates quite considerably over short distances, say a few miles. I would like to ask Mr. Hurst if these fluctuations would appear in his results, or would they be lost amongst the effects of aircraft pitch and roll, which must be smoothed out?

Dr. R. C. Sutcliffe: I think it should go on record that these jet-stream cross-sections are the first to be made on the basis of observations at short-space intervals. All the studies so far made have depended on the network of aerological stations rarely less than 200 mi apart and the detailed structure has been largely conjectural. Frequently when a new technique is introduced to give us new facts we discover unsuspected behaviours. In this case however we seem to get only confirmation of what we had previously inferred. However, the fact that a smooth profile is consistent with the estimated errors of observation is no proof that the true profile is in fact smooth. One wonders whether by more refined statistical methods it might be possible to disentangle the errors from the real variations which probably exist.

This technique may have little more to tell us from regions where we have a good aerological network, but it might be the best way of studying conditions (for research purposes at least) in regions where the establishment of an aerological network is economically prohibitive.

Mr. O. Neumark: Could Mr. Hurst tell us whether the aircraft was flown manually or with an automatic pilot? Would not the use of the latter method reduce the wind errors? Would it not be well worth while to use gyro-stablilized cameras, or at least gyro-stabilized and collimated earth vertical graticules on each photograph, in order to eliminate all errors entirely and thus provide a true picture of the fine structure of the wind without smoothing?

Mr. J. Durward: In his reference to the possible elimination of Ocean Weather Ships, did Mr. Hurst imply that aircraft reconnaissance of the type he describes could take their place? Could the method be used over the sea?

Mr. G. W. Hurst (*in reply*): I have the good fortune to have supporting me this afternoon Fg. Off. F. N. Slaymaker, the pilot of the two jet stream flights, who will be prepared to answer any points on flying which Fellows may wish to raise.

It is difficult by this photographic technique to distinguish between irregularities due to aircraft unsteadiness and those due to real wind variation, and consequently in reply to Dr. Brewer, it is problematical whether small-scale fluctuations, even though they be of the amplitude of a few miles, which are evident by camera-obscura examination, could be identified by normal overflying photographic methods. As Dr. Sutcliffe points out, refined statistical methods may possibly distinguish between flying errors and real variation, but a suitable sieve will not be easy to evolve. There may be the possibility of building up a future profile from above by radar, and not by direct ground, photographic methods; if this could be done satisfactorily, many of the possible errors discussed in my paper could be eliminated at source.

Dr. Sutcliffe's kind remarks regarding the technique as a whole are much appreciated and, as he says, it may well be helpful to arrange flights over areas where the reporting network is sketchy, and, for example, in the event of the withdrawal of the weather ships. In reply to Mr. Durward, the last remark is not made as a serious suggestion that an aircraft flying at a great height could provide the same information as weather ships on their stations with their detailed instrumental surface and upper-air measurements, but they could certainly constitute a very valuable auxiliary weapon.

Fg. Off. F. N. Slaymaker (*in reply*): In reply to Dr. Scorer's question, I did notice some clear-air turbulence on the second flight from the Firth of Tay area to a point about 20 mi south of Cape Wrath; the turbulence was at no times severe, and could best be classified as 'cobble-stones.' I have no clear recollection of any turbulence on the first flight. (The photographs

themselves would support a certain unevenness over north Scotland on the second flight, but there was no evidence of unevenness on the first flight at any stage. G.W.H.)

In reply to Mr. Neumark, the aircraft was flown manually throughout, and I think that careful manual flying will be nearly as steady as flight controlled by an automatic pilot, since the latter only corrects after the displacement has occurred. I have had no experience with gyro-stabilized cameras, and cannot comment usefully on this point, except to add that I believe there must be still some precession even after gyro-stabilization. There are however possibilities of obtaining more accurate positioning by the use of Decca equipment; in this case, the position is continuously presented with a high degree of precision (a few yards from 30,000 feet), and the instrument could be recorded photographically in flight.

551.506.7 (261) (4)
551.557.35 : 551.547

A further study of the upper westerlies; the structure of the wind field in the eastern North Atlantic and western Europe in January 1950

By D. H. JOHNSON

(Read 18 November 1953. See *Q.J.*, **79**, p. 402)

Mr. L. C. W. Bonacina : I should like to ask Mr. Johnson whether it is a fact, as I have read that it is, that the most powerful jet streams in the northern hemisphere blow in latitude 30°, in winter at a little over 30,000 ft. If this is really the case it explains why Mt. Everest expeditions have been restricted to the brief interval between the departure of the upper westerlies and the arrival of the south-west monsoon. It also suggests that there may be a correlation between these times of departure and arrival which would be useful for forecasting the Indian rains. With modern means of sounding the atmosphere it should not be difficult to ascertain whether such a correlation exists.

Mr. J. S. Sawyer : Although numerous atmospheric cross-sections have recently appeared in the technical literature, few of them refer to longitudes such as that of western Europe which intersect a semi-permanent ridge in the normal upper-flow pattern of the hemisphere. The labour which Mr. Johnson has expended in producing the sections which he has described to us was undoubtedly well worthwhile.

Instr. Cdr. C. R. Darlington : Analysis of records for four years from Ocean Weather Ship *Mike* (66°N, 2°E) shows that the frequency of high winds above this station varies very little in the three ' dark ' seasons – Autumn, Winter and Spring – but that the frequency decreases markedly in Summer. The maximum wind speeds occur at about 30,000 ft at all seasons, which corresponds with the height of the wind maximum shown on Mr. Johnson's mean cross-section from Angmagssalik to Tunis. The position of *Mike* is almost exactly upwind (following the direction of the mean contours) from the position of Mr. Johnson's mean wind maximum.

Mr. D. H. Johnson : In reply to Mr. Bonacina, it is now well established that a strong jet stream flows across North Africa and Asia in winter in latitudes 20°N to 30°N, centred at a height of about 40,000 ft. The question of the suggested association between the time of departure of the westerly jet from northern India and the burst of the monsoon is an interesting one. M. T. Yin (1949) has expressed the belief that the two events coincide. R. Frost (1953) however, from a study of the upper-air circulation over Arabia, suggests that the onset of the monsoon precedes the movement of the jet stream to a position north of the Himalayas.

References

Frost, R. 1953 ' Upper air circulation in low latitudes in relation to certain climatological discontinuities,' *Prof. Notes Met. Office*, London, **7**, No. 107.

Yin, M. T. 1949 ' A synoptic-aerologic study of the onset of the summer monsoon over India and Burma,' *J. Met.*, **6**, p. 393.

551.557.2 : 551.547

Transverse flow at entrance and exit to jet streams

By R. MURRAY and S. M. DANIELS

(Read 18 November 1953. See *Q.J.*, **79**, p. 236)

DR. J. M. STAGG : This is a most satisfying paper. We all know that when air is accelerated it is flung to the left and when it is decelerated it is flung in the opposite direction, and this is one of the factors that have to be taken into account in a qualitative way when looking for areas of likely convergence and divergence near the surface. But in this paper the authors have given us for the first time the actually observed measures and the distribution of the cross-flow in those important regions where fast-flowing currents are found. To me it would have added still further to the interest of the paper if the authors had used their estimates of the dimensions of the funneling into and out of the jet to calculate what the transverse flow should be so that we might compare their observed quantities with theory. I have also wondered whether the quite considerable accelerations of both sign that the authors have shown to occur in association with jet streams may not contribute to the kind of turbulence which aircraft experience in those regions. May it not be that there are factors arising from these accelerations and the induced circulations which are not taken account of by the rather coarse measurements of sheer in the vertical and in the horizontal which have been used in trying to find correlations with turbulence near jet streams ?

MR. C. E. WALLINGTON : As Dr. Stagg has remarked, the paper supports the simple conception of ' acceleration—to the left.' But this seems to apply only at the level of the jet axis. According to the paper the transverse flow is reversed at about 150-200 mb below the jet axis. Is this apparent reversal due to the method of investigation, or is it real ? If it is real it may be rather difficult to justify theoretically.

DR. R. C. SUTCLIFFE : In dynamical meteorology it is an event if theory can predict something which is later confirmed by observation. Even when the theory is the simple relationship between ageostrophic wind-component and acceleration we are impressed when statistics reveal the connection so clearly as in this paper. The magnitude is obviously about right but it would have been worth while for the authors to check quantitatively : they say nothing about the scale of motions which they are studying.

Although the wind at jet level does in general flow through its pattern, the flow-system generally moves in the same direction so that the true acceleration is less than that given by assuming a stationary flow pattern. At some lower level the pattern normally moves with the mean wind speed and there will be little acceleration. This level, which would roughly coincide with the level of non-divergence, is indicated in the statistics by the zero mean value in the cross flow at 150-200 mb below the jet level. Below this level theory would predict the opposite sense of cross-flow, again as confirmed. One would also expect another zero to be reached above the jet level but perhaps observations are too few to demonstrate this clearly.

MR. J. M. CRADDOCK : This paper shows that at the entrance and exit of a jet stream the existence of an ageostrophic wind component can be proved by a comparison of observed winds with the run of the upper-air contours. This result raises the prospect of success of similar investigations into other regions where ageostrophic winds are suspected, for example, in association with warm fronts.

MR. R. MURRAY and MISS S. M. DANIELS (*in reply*) : As Dr. Stagg has stated, it is of some interest to know whether or not the dimensions of the space variation of flow downwind into and out of the jet stream are such as might produce the observed mean transverse flow. For reasons which have been mentioned in the paper, there is a good deal of uncertainty in estimates of transverse flow in individual cases; the standard errors of estimates of the mean values near the wind maximum are fairly large, so that there is doubt about the precise magnitude of the mean transverse flow but reasonable certainty about the direction of the mean cross-flow. On the assumption that the jet-stream pattern is stationary and that the only acceleration term is the one arising from the ' confluence ' or ' difluence ' in the wind field, then it is clear that the transverse flow could be two or three times the mean values of this paper. In practice, however,

there are other terms in the equations of motion which counteract the term arising from acceleration along the flow. We find it difficult to picture the physical relationship between the occurrence of turbulence near jet streams and the type of cross-jet vertical circulations discussed. Certainly it is a fact of observation that clear-air turbulence frequently occurs near jet streams in places well away from the entrance or exit regions, so we think it is unlikely that there is a close relationship between turbulence and transverse and vertical circulations of the type we have discussed. Nevertheless, it is just possible that fluctuations in the magnitude of the acceleration near the entrance might lead to very-small-scale wave-like perturbations running rapidly downwind through the jet stream pattern, and that these perturbations might be relevant to the turbulence problem by causing short-lived, localized variations in horizontal and vertical shear – but this is speculation.

In reply to Mr. Wallington we would like to say that it appears fairly certain that there is a real difference between the values of the mean transverse flow at the jet level and at levels below (the statistical significance of the data has been discussed in the paper). There seems little doubt that the mean transverse flow at 150-200 mb below the jet level is quite small, although in individual cases the zone of zero or reverse cross-flow probably varies considerably. Furthermore, a reversal of the transverse flow must surely occur somewhere in the middle troposphere for it is well established theoretically that convergence in the lower troposphere must be roughly balanced by divergence in the upper troposphere, and vice versa.

Dr. Sutcliffe has neatly and lucidly summarized the dynamical background of relevance to the statistics. The jet streams investigated varied considerably in longitudinal extent, perhaps averaging 1,000 to 1,500 mi in length. Incidentally, many of the jet streams did not have both entrance and exit as pronounced features, although either the entrance or the exit was required to lead to or from a zone of quasi-parallel flow of small curvature; well upwind or downwind from the fairly straight flow there was sometimes considerable curvature in the pattern. As regards the possibility of another zero in the transverse component above the jet level, the highest observations were taken at 100 mb above the wind maximum since this was the highest level at which a sufficiently large statistical sample could be obtained : in very strong winds the observations are frequently missing above the 150 mb which roughly corresponds to the topmost level employed in the paper.

We agree with Mr. Craddock that a statistical investigation of ageostrophic winds in other synoptic types might be profitable when there is considerable uncertainty in individual cases.

Proceedings at meetings of the Society

Ordinary Meeting, 21 October 1953

The following candidates were declared to have been elected by the Council during the summer recess :

Fellows

Birch, Fg. Off. Neville Hamilton, 22 St. Agnes Road, Moseley, Birmingham.
Brumby, Capt. Peter Allan Keith, c/o West African Airways Corporation, Airways House, Lagos, Nigeria.
Cleaver, Walter William, 148 Lawford Road, Rugby, Warwickshire.
Dickson, John Lawrie Christie, Willowbank, The Loan, South Queensferry, West Lothian, Scotland.
Egelstaff, John Hugh, ' Maristowe,' Burlington Road, Burnham, Bucks.
Emore, Isaac Oti, B.Sc., 48 Park Road, New Barnet, Herts. Permanent address : Meteorological Office, Lagos, Nigeria.
Greves, Stanley Southam Hyla, 34 Welldon Crescent, Harrow, Middlesex.
Hawney, Mrs. Anna, Parsons Mead, Ashtead, Surrey.
Jeffree, Edward P., B.Sc., West Lodge, Danestone, Woodside, Aberdeen.
Jones, Instr. Lieut. Oscar Graham, R.N., B.A., Caelo, Church Lane, Hooley, nr. Chipstead, Surrey.
National Institute of Oceanography, Wormley, Nr. Godalming, Surrey. (Nominee : The Director. *At present*, Dr. G. E. R. Deacon, F.R.S.).
Phillips, Percival Edmund, B.Sc., Bowershall, By Dunfermline, Fife.
Randolph, Major Peter, 61 Park View Road, Ealing, London, W.5.
Satow, Cdr. Christopher Paul, R.N., Knockhill, West Kingsdown, Sevenoaks, Kent.
Warrell, Alan, B.A., 57 Edge Hill Court, Wimbledon, London, S.W.19.

Foreign Members

Ivey, Major Harold E., Detachment 2, 29th Weather Squadron, A.P.O. 616, c/o Postmaster, New York, N.Y., U.S.A.
Pellino, Joseph, Panair do Brasil, Caixa Postal 694, Rio de Janeiro, Brazil.
De Sant'ana, Jose Alves, Panair do Brasil, Dept. Meteorologia, Caixa Postal 694, Rio de Janeiro, Brazil.
Smagorinsky, Joseph, Ph.D., R.D.5, Box 512A, Alexandria, Virginia, U.S.A.
Smith, Minard Eugene Eaton, M.A., Technical Services, Meteorology Branch, Dugway Proving Ground, Tooele, Utah, U.S.A.
Tepper, Morris, M.A., Ph.D., U.S. Weather Bureau, Washington 25, D.C., U.S.A.

Student Member

Smith, Christopher Cyril, Railway Hotel, Woodhall Spa, Lincs.

The following candidates were then declared to have been elected that day by the Council :

Fellows

Alexander, Geoffrey Newman, Bayview Road, Belgrove, Victoria, Australia.
Blydenstein, Elisabeth Rosamunde, 6 Cornwall Gardens, London, S.W.7. (*transfer from Student Membership*).
Boyd, Trevor Dempster, B.Sc., ' Skibbereen,' Cultra, Holywood, Co. Down, Northern Ireland.
Byron-Scott, Roland Alexander David, c/o The Vicarage, Masham, Nr. Ripon, Yorks.
Gillam, William Thomas, 21 Gloucester Place, London, W.1. (*Life Member*).
Hare, Archie Winter, M.A., 16 Ivy Road, Moorhead, Shipley, Yorks.
Harris, Philip Arthur, B.Sc., 64 Woodside Avenue, Finchley, London, N.12.
Jolley, William, ' Dorville,' Leylands Road, Burgess Hill, Sussex.
Knight, John, B.A., Tamarisk, The Avenue, Kingston, Lewes, Sussex.
Lloyd, Donald Charles, M.A., Tolpits House, Tolpits Lane, Watford, Herts.
Neave, Charles Frederick, c/o 25 Coquet Street, Jarrow, Co. Durham.
Newcomb, Thomas Peter, M.Sc., 2 Oakland Terrace, Long Eaton, Nr. Nottingham.

Foreign Members

Mordy, Wendell A., B.A., Pineapple Research Institute of Hawaii, P.O. Box 3166, Honolulu, Hawaii.
Sekera, Zdenek, Department of Meteorology, University of California, Los Angeles 24, U.S.A.

ASSOCIATE

HARDEN, Donald, Birkdale Road, Birkenhead, Auckland, New Zealand.

STUDENT MEMBERS

BERRY, Grace, 84 Abbotsford Road, Liverpool 11.
CRAGGS, John Strong, Station House, Alston, Cumberland.
CURTIS, Winifred Joan, 57 Holland Road, East Ham, London, E.6.
CUSDIN, Geoffrey, 252 Carlton Road, Gidea Park, Essex.
ERASMUS, David Harold, 10 Trelawny Avenue, Flint.
FRETTSOME, Eileen M., 59 Nutfield Road, South Merstham, Surrey.
GREENWOOD, Erica, 20 Lauriston Road, Wimbledon, London, S.W.19.
KEMP, Margaret G., 15 Harecroft Road, Otley, Yorks.
LEFEVRE, Susan Mary, 1 Lady Hay, Worcester Park, Surrey.
PEUNGIEZ, Julian Patrick, Grange House, Alston, Cumberland.
RAYNER, Margaret Jean, 31 Church Avenue, Pinner, Middlesex.
SKINNER, Delia Joan, 149 Gunnersbury Park, Pope's Lane, Ealing, London, W.5.
TAYLOR, John Brian, Byways, Stapeley, Nantwich, Cheshire.
TURNER, L. Joy, 101a Speldhurst Road, Bedford Park, Chiswick, London, W.4.

Mr. E. L. Deacon of the Meteorological Physics Section, Commonwealth Scientific and Industrial Research Organization, Australia, gave a short talk on the work of his Department.

The following papers were read and discussed :

(*a*) ' The conditions for dynamical similarity of motions of a frictionless perfect-gas atmosphere ' by G. K. Batchelor (*Q.J.*, **79**, p. 224).

(*b*) ' A quantitative analysis of the evolution of large-scale flow with regard to the effect of eddy motion ' by F. A. Berson (*Q.J.*, **79**, p. 210). The paper was presented by Mr. J. S. Sawyer.

(*c*) ' On the theory of large-scale disturbances in a two-dimensional baroclinic equivalent of the atmosphere ' by P. D. Thompson (*Q.J.*, **79**, p. 51). The paper was presented by Mr. F. H. Bushby.

Ordinary Meeting, 18 *November* 1953

The following candidates were declared to have been elected that day by the Council :

FELLOWS

CANTLE, Wilbert Graham, 3 Falcon Close, Westbury-on-Trym, Bristol.
CHAMBERS, Ernest, 592 Staines Road, Bedfont, Middlesex.
CLARKSON, Leo S., 17 Kynaston Close, Harrow Weald, Middlesex.
HITSCHFELD, Asst. Prof. Walter, Ph.D., Department of Physics, McGill University, Montreal, Quebec, Canada.
MULLINEUX, A. Rodney, c/o Meteorological Office R/S, R.A.F. Leuchars, Fife.
MURDEN, Peter M.Sc., A.R.C.S., 189 Sirdar Road, Wood Green, London, N.22.
ROWAN, Flt. Lt. Maurice, M.B.E., R.A.F.V.R., 55 Tintern Avenue, Westcliff-on-Sea, Essex.
TAYLOR, Instr. Lt. Cdr. John E., M.A., R.N., c/o D.E.D., Admiralty, London, S.W.1.
THATTE, Raghunath, Department of Natural Philosophy, The University, Drummond Street, Edinburgh 8.

FOREIGN MEMBER

HLA, U., B.Sc., Rangoon Airport, Mingaladen, Burma.

STUDENT MEMBER

SAUNDERS, Peter Michael, B.Sc., A.R.C.S., 7 Rushout Avenue, Kenton, Harrow, Middlesex.

The following papers were read and discussed :

(1) (*a*) ' The profile of a jet stream observed 18 January 1952,' by G. W. Hurst (*Q.J.*, **78**, p. 613).
(*b*) ' The profile of a jet stream observed 1 September 1952 ' by G. W. Hurst (*Q.J.*, **79**, p. 407).

(2) ' A further study of the upper westerlies ; the structure of the wind field in the eastern North Atlantic and western Europe in January 1950 ' by D. H. Johnson (*Q.J.*, **79**, p. 402).

(3) ' Transverse flow at entrance and exit to jet streams ' by R. Murray and S. M. Daniels (*Q.J.*, **79**, p. 236).

Third Annual Dinner, 20 November 1953

The third Annual Dinner of the Society was held on 20 November 1953 at the Hotel Rubens, London. Some 55 Members and guests were present, the President, Dr. O. G. Sutton, C.B.E., F.R.S., being in the chair. The guest of honour was K. W. M. Pickthorn, D.Litt., M.P., Parliamentary Secretary to the Ministry of Education, who replied to the toast ' Education in Science ' proposed by the President. Other guests included the Chairman of the Committee of Vice-Chancellors and Principals of the Universities of the United Kingdom, the Chief Education Officer of the L.C.C., the President of the Headmasters' Association, the Editor of *The School Science Review* and the Director of the Council for the Promotion of Field Studies.

A full account of the proceedings was published in the December 1953 issue of *Weather*.

Popular Lecture, 2 December 1953

Mr. R. G. Veryard, B.Sc., being in the chair, Mr. J. S. Sawyer, M.A., gave the annual popular lecture on ' Weather forecasting,' in the Lecture Theatre, Royal Institution. The lecture was open to sixth-form students, of whom 430 were present by arrangement with the education authorities of the London, Buckinghamshire, Essex, Hertfordshire, Middlesex and Surrey County Councils.

Ordinary Meeting, 16 *December* 1953

The following candidates were declared to have been elected that day by the Council :

FELLOWS

CLIFTON, Capt. Denis Percival, 1 Derwent Drive, Hayes, Middlesex.
CUNNINGHAM, Capt. John Peter, P.O. Box 4323, Nairobi, Kenya.
CURTIS, Ernest William, 7 Beech Close, Hamble, Hants.
DATAR, Madhav Gopal, Nautical & Engineering College, Azad Maidan, Bombay 1, India.
EDWARDS, Ronald Edgar, 14 Leaside Way, Southampton.
GRAHAM, James Ronald, M.A., 25 Restalrig Avenue, Edinburgh 7 (*transfer from Student Membership*).
LIU, Peter Yu Ling, Royal Observatory, Hong Kong.
NICOL, William Stewart, 11 Shetland Road, Westbury-on-Trym, Bristol.
ROBERTS, Charles Wulstan, Goderich, Sarisbury Green, Nr. Southampton, Hants.
WEST, Instr. Lt. Peter Raymond, B.Sc., R.N., 60 Boldshay Street, Barkerend Road, Bradford, Yorks.
WICKHAM, Peter Guy, B.A., Meteorological Office, R.A.F., Lindholme, Nr. Doncaster, Yorks.

The meeting took the form of a joint discussion with the Royal Astronomical Society on ' Stellar scintillation.' Some 100 Members and visitors were present, and the opening speakers were :

DR. H. E. BUTLER (Royal Observatory, Edinburgh), DR. M. A. ELLISON (Royal Observatory, Edinburgh), DR. E. C. S. MEGAW (Director of Physical Research, Admiralty), DR. J. VAN ISACKER (Royal Meteorological Institute of Belgium).

Reviews

Deep-sea Research. Published quarterly, annual subscription £4. 10s. 0d.

We note the appearance of Vol. I, No. 1, of a new scientific journal, *Deep-sea Research*. Published by the Pergamon Press, and edited by L. Fage, C. D. Ovey and M. Sears, a field is covered where meteorology, oceanography, geophysics, and other sciences meet. We wish the new venture all success.

T.W.W.

Advances in Geophysics, Vol. I. Edited by H. E. Landsberg. New York, Academic Press Inc. (Academic Books Ltd., London), 1952. Pp. xi, 362. $8.50.

This is the first of a series of occasional publications by means of which Dr. Landsberg hopes to produce a cross-fertilization between closely allied fields, a process otherwise hindered by the widely scattered literature. Some increase in specialization is to be expected in the various branches of geophysics as they develop but the reviewer feels strongly that they also have much to gain by observing each other's techniques and the models they work with, and so he is indeed ready to welcome Dr. Landsberg's series.

Two contributions to this first volume appear to be particularly successful, that on 'Automatic processing of geophysical data' by J. C. Bellamy, and a second on 'Some new statistical techniques in geophysics' by A. Court. The first of these subjects makes an obvious high claim on our attention for we spend great effort on the design, manufacture and use of geophysical apparatus – a synoptic reporting station is incidentally a part of this apparatus – but so far no corresponding effort has been made to provide for an efficient extraction of the physical, or merely of the utilitarian content of the data, or – a step in that process – to give them effective presentation. Punched cards, aerological diagrams and synoptic charts abound, it is true, but these are only some of the possible, and possibly less efficient aids which Dr. Bellamy suggests might be used. With electronic computation of weather forecasts only just round the corner, the problem of the assimilation of synoptic data, or of data based on them, by these computing machines would already appear to be rather urgent. The digestion of upper-air data for physical climatology poses a problem of equal insistence. Although the geophysical laboratories of the future may not be quite the array of analogue and digital recorders and computers which Dr. Bellamy puts before us, there is no gainsaying the timeliness of his discussion.

Dr. Court is also concerned to show us how more or less recent advances, here in the field of statistical theory, will enable us to make more use of our data. Thus, he applies new knowledge to the examination of bi-modal distributions as may occur, for example, with tropopause height, to the problem of extreme values which must loom large in the calculations of the public-works engineer, and to the properties of circular variables. Altogether an illuminating example of how 'out of this nettle, danger' – for statistics is still something of a nettle to many of us – to pluck a flower of worth.

The other contributions can be little more than listed. Bolin writes on the general circulation and discusses in a stimulating way the interplay between barotropic and baroclinic behaviour of the atmosphere; a particularly enlightening article for the looker-on in this field. There are two articles on the high atmosphere, one by F. L. Whipple on the interpretation of meteoritic data and the other of nearly 100 pages by N. C. Gerson on unsolved problems. The latter is disappointing to at least one reader. It is diffuse and somewhat obvious, abounding in sentences like 'A determination of temperature from an examination of solely radiative equilibrium seems too restrictive to be applied to the atmospheric shells unless it can be shown that the effects of conduction and advection are negligible.' But it does bring out the uncertainty attaching to estimates of temperature based on the assumption of thermodynamic equilibrium. Other geophysical fields are represented by articles on estuarine hydrography, on gravity and the exploitation of gravity measurements in geodesy and geology, and on aeromagnetic surveying. There is also a complete author and subject index which is not the least valuable feature of the volume.

Dr. Landsberg is to be congratulated on a generally very successful first volume of his series. The price is probably rather high for the pockets of most readers on this side of the Atlantic and one wonders whether a book in a rather less finished, stylish form could be produced at a substantially lower figure. Active workers in geophysics would certainly do well to put it on their shelves and would probably be ready to forgo appearance while retaining utility.

P.A.S.

Modern developments in fluid dynamics : High speed flow. Edited by L. Howarth, Oxford (Clarendon Press), 1953. 2 vols., pp. xvi, viii, 875; 26 plates, 288 Figs., 13 tables. 84s.

This important work is a companion to the two volumes under the same main title edited by S. Goldstein (1938) which dealt chiefly with incompressible flow. The present work is concerned principally with the effects of compressibility at both subsonic and supersonic speeds and it includes accounts of many of the important theoretical and experimental advances made during and since the last war.

It is a collective work prepared under the aegis of the Fluid Motion sub-committee of the Aeronautical Research Council and is edited by L. Howarth with the assistance of H. B. Squire and the late C. N. H. Lock. Unlike the course adopted in the earlier volumes, the authors of each chapter are named individually

and read as follows : Introduction (L. Howarth), The equations of flow in gases (L. Howarth), The method of characteristics (R. E. Meyer), Shock waves (C .R. Illingworth), Blast waves (G. J. Kynch), Some exact solutions of the equations of steady homentropic flow of an inviscid gas (W. G. Bickley), One-dimensional flow (O. A. Saunders), The hodograph transformation (M. J. Lighthill), Approximate methods (G. N. Ward), Unsteady motion (G. Temple), Boundary layers (A. D. Young), Experimental methods (D. W. Holder, D. C. Macphail, J. S. Thompson), Flow past aerofoils and cylinders (W. A. Mair, J. A. Beavan), Flow past bodies of revolution (W. F. Cope), Heat transfer (H. B. Squire) (this is a revision of the chapters in the 1938 volumes and is placed more appropriately in the new volumes dealing with high-speed flow). As well as giving some idea of the subject matter treated, the above list will show the strength of the team responsible for the work and give a guarantee of its worth.

From the extremely well-written introductory chapter right through to the comprehensive index, these two volumes contain important subject matter written by experts in their own fields and welded into a homogeneous account by skilled editing. They are as up to date as is possible for a work of this magnitude on a subject which is advancing so rapidly.

Lack of space forbids a detailed discussion chapter by chapter and a few general remarks must suffice. These volumes deal with a subject which does not make easy reading. It is, however, a subject which is of great importance today and one in which the results of recent research are scattered through many scientific journals. Giving as they do a connected account and being such a valuable source of references, the volumes under review will surely become classics and will be found alongside their earlier companion volumes in many bookshelves throughout the world.

Although a great deal of the subject matter is theoretical, the practical aerodynamicist will find much in this work which will interest him. The long chapter on experimental methods seemed to the reviewer to be a most useful one and the large number of diagrams and the many beautiful plates are valuable features. No extensive data for the calculation of compressible flows are included but this information can be found in two further companion volumes prepared by the Compressible Flow Tables Panel of the Aeronautical Research Council.

Professor Howarth and his team of authors are to be congratulated on producing this work at such an opportune moment and so too is the Clarendon Press for a production which is well up to its usual high standard.

C. J. Tranter.

The scientific work of René Descartes (1596-1650). By J. F. Scott, London (Taylor and Francis), 1952. Pp. 211; plate, Figs. £1.

Some scientists are outstanding for their individual discoveries : others for the tools they provide which their fellows can wield. Descartes, while he did make significant discoveries, owes his permanent eminence in science to his analytical tool – coordinate geometry. It is now, however, so much taken for granted that meteorological text-books contain no explicit acknowledgment of their debt to Descartes, without whose tool dynamical, thermodynamical and micro-meteorology could not have developed. Any explicit acknowledgment of Descartes' contribution to our science which text-books do contain is limited to a reference to his explanation of the rainbow : correct as far as it could be at a time when the variation in refrangibility of light of different wavelengths was unknown.

The present work by Dr. Scott gives an admirable account of Descartes' scientific work with some brief, but unavoidable, notes, on his philosophy and his religious beliefs. Dr. Scott does not limit himself to setting forth what Descartes did, but relates it to the work of his predecessors and contemporaries and brings out very clearly the truth that a scientist's discoveries are inevitably the result of cumulative knowledge and that the names of many forgotten heroes might be not unfairly associated both with the explanation of the rainbow and with Descartes' developments in mathematics.

There are thirteen chapters in the book which has an illuminating Foreword by Professor Turnbull and a useful Appendix ' Brief Biographical Notes on some of the Less Important Persons mentioned in the preceding pages ' The cynic, without real justification, may growl that apparently in science too L.I.P.'s get the publicity; for myself, the capitals provoked a smile.

Descartes' main published works were *Discours de la méthode plus La Dioptrique, Les Météores et La Géométrie*, 1637; *Meditationes de Prima Philosophia*, 1641 and *Principia Philosophiae* 1644. Dr. Scott devotes Chap. III to the Discours, IV to the Dioptrique, V to the Météores, V, VI, VII, VIII and IX to the Géométrie and X, XI, XII to the Principia.

Meteorologists will naturally be specially interested in Chapter V, ' Les Météores.' This gives a preliminary outline of pre-Descartes meteorology and of Descartes' ideas on the constitution of matter and of light and heat. Dr. Scott emphasizes that though, contrary to what has been sometimes alleged, Descartes did in fact do experimental work and frequently suggested experiments to confirm his conclusions, he considered they were limited to this confirming function. If they failed to confirm his conclusions, then the experiments were at fault; ' few people are capable of doing experiments well. Often through doing them ill, one finds the contrary of what one should find.' Meteorologists know this. Indeed they are occasionally too Cartesian towards observations.

As Dr. Scott says, the outstanding feature of Les Météores is Descartes' explanation of the formation of the rainbow by refraction and internal reflexion in the raindrops, and his computation of the radii of both

the primary and secondary bows. It came as a surprise to me that Descartes should have known that the refractive index was less for warm water than for cold. But his remark, quoted without comment, that this ' may increase the radius of the primary bow by one or two degrees at most ' allows too liberal a margin. A change of temperature from 40°F to 95°F would increase the radius by only 20 minutes and this is the maximum as nowhere is the temperature of rainbow rain likely to exceed 95°F. (A change of ·0010 in the refractive index changes the radius by $8\frac{3}{4}$ minutes. There are errors in the table of radii for the different Fraunhofer lines in Pernter's *Optics*, p. 521, 1910.) Descartes' value for the mean radius of the primary bow, 41° 30′ corresponds with a refractive index 1·3370, the value for the F line.

In view of the note on p. 8 that Descartes' interest in meteorology had been stimulated by an observation of parhelia by Scheiner at Frascati on 20 March 1629, his attempt at an explanation of coronae and haloes might well have received more than the brief mention on p. 81 which gives no indication of the nature of the explanation beyond the statement, ' they are the result of unusual combinations of the ordinary causes of the phenomena which give rise to unusual results '; not very illuminating.

The chapters on geometry give a very convincing demonstration of the mathematical ability of Descartes; and more than once Dr. Scott emphasizes and illustrates the outstanding importance of the link forged by Descartes between algebra and geometry. Descartes' rule of signs (which I was interested recently to see used in a paper by the late L. F. Richardson) is given by Dr. Scott as ' In any equation there can be as many true roots as there are changes of sign and as many false roots as there are two like signs following.' True and false mean real positive and real negative roots respectively. A preferable form of the rule for negative roots, given by Burnside and Panton, is ' the number of negative roots cannot exceed the number of changes of sign in the equation derived by substituting $-x$ for x.' In the form given by Dr. Scott the negative rule may not apply to an incomplete equation whereas the B. and P. form does, as may be seen for example, in the equation $x^4 - 5x^2 + 4 = 0$ which has two real positive and two real negative roots.

The Principia in so far as it is an attempt to explain the structure of the universe on the basis of a system of vortices is now mainly of historical interest. But to some political thinkers it stands out as the *fons et origo* of the dilemma into which duality has plunged human civilization. But the duality exists quite independently of its explicit formulation by Descartes; indeed Joanna Field, almost in despair, but ' remembering Descartes,' served seven years in the exploration of this duality and found satisfaction. No doubt Descartes' formulation, like his explanation of the rainbow, is incomplete. In the living world, as often in meteorology, there may well be no sharply defined ' front ' between two antithetical entities, as Descartes supposed, but an ill-defined zone which partakes of the character of both; without however nullifying their distinctive character in regions distant from this zone.

In the final chapter Dr. Scott summarizes Descartes' main achievements and describes his personal failings – his egoism, coldness and inability to recognize the value of the work of contemporary mathematicians and natural philosophers such as Galileo, Pascal and Fermat; failings regrettable but trivial in comparison with his great achievements and his vast and unwearied labours in the cause of science. His comparatively early death was, in fact, due to his braving the rigours of life in winter in Stockholm to support the Queen of Sweden's design of encouraging learning in her realm.

Descartes wrote ' my third maxim was that I would strive always to conquer myself rather than fortune and to change my own desires rather than alter the World Order. I suppose that is, in fact, the way ' to change the World Order. He did it.

E.G.

Waves and Tides. By R. C. H. Russell and D. H. Macmillan. London (Hutchinson's Scientific and Technical Publications), 1952. Pp. 348; 17 plates, 102 Figs., tables, appendices. 25s.

This book is intended for ' all who are seriously interested whether for business or pleasure ' and the authors are thus committed to writing a non-technical account of a very technical subject giving references to original papers for those readers who wish to pursue the subject further. Mathematical reasoning has been avoided as far as possible. The book is divided into two independent parts, ' Waves ' by Mr. R. C. H. Russell, and ' Tides ' by Commander D. H. Macmillan.

Mr. Russell has a difficult task. His subject comprises at present an intricate mathematical theory of low waves in windless areas, a few basic experiments among which Bagnold's work is outstanding, and a large number of wave records which (it is hoped) may come in useful some day. The correlation of theory and experiment, on which the progress of science depends, has been carried out in very few cases, as, e.g., in the work on wave propagation and on microseisms undertaken at the Admiralty Research Laboratory during the last ten years, and in the work on wave diffraction and refraction undertaken by the University of California. The fundamental problem of wave generation is not yet understood, although empirical rules allow wave prediction from weather charts to a fair degree of accuracy. Mr. Russell is the first writer to attempt a full account of this subject.

There can be few readers whom this book can teach nothing. With its wide scope it would not be reasonable to expect complete accuracy in every part and there are in fact errors but their number is not unduly large. On p. 46 the concept of group velocity is introduced. In deep water the kinetic energy is equal to the potential energy, and the total energy is equal to their sum. ' It is the kinetic energy that stays still and the potential energy that moves along at the speed of the waves ' and hence a group advances with half the phase velocity. The statement about the energies (which is taken from earlier accounts) is in

my opinion almost meaningless and is bound to discourage the reader. The alternative discussion on p. 163 is much better from this point of view. In the geophysical application moreover the group velocity arises not as the velocity of propagation of a long group of waves of a definite period, but as the velocity with which a given wave period spreads from a centre of disturbance (this idea also underlies the most satisfactory mathematical treatment of group velocity, Jeffreys and Jeffreys, *Mathematical Physics*, §17.08, 1946). The ARL spectrum analysis of sea waves, only briefly mentioned in this book, showed excellent agreement between theory and observation. If Mr. Russell can devise a good non-mathematical presentation of group velocity he will have rendered a service to science. In the discussion of breakers (pp. 76-80) it is suggested that the asymmetric development of waves near the beach is due to the slope : the front of the wave is in shallower water than the rear, and thus travels more slowly; it is known, however, that very long waves tend to become asymmetrical even in water of uniform depth, but this effect is not discussed. (For references and further details see a paper in *Proc. Camb. Phil. Soc.*, **49**, p. 685, 1953.) It is not known how far this effect is important in comparison with friction and backwash from the beach. The section on diffraction (pp. 90-94) contains several doubtful statements. It is asserted that waves in a harbour can be reduced by reducing the width of the entrance, but this is not borne out by the experiments of McNown (' Gravity waves,' NBS circular No. 521, p. 153, 1953). The objections on p. 93 to the analogy between water waves and light waves (or, better, sound waves) are unfounded. When the sound waves are two-dimensional the analogy is complete; (see Penney and Price, *Phil. Trans.* A, **244**, p. 236, 1952, where the equations governing the two wave types are seen to be identical). A mathematical solution for oblique incidence on a semi-infinite breakwater does exist (Penney and Price, *ibid.*, p. 243). Huygens's principle, however, cannot be usefully applied to the mathematical functions occurring in that solution, for the reason stated by Mr. Russell (p. 93). As for the last statement on p. 93, it is to be noted that the analogy involves *two-dimensional* sound waves. The discussion of particle drift (pp. 109-112) is based on a suggestion by N. F. Barber; it has been shown since by M. S. Longuet-Higgins (*Phil. Trans.* A, **245**, p. 535, 1953) that the effect of viscosity is more radical than had been thought, and he obtains results consistent with Bagnold's experiments, although a complete treatment of this problem has not yet been found practicable. The last equation on p. 161 of Mr. Russell's book does not follow from the last but one; for d/H small, the last equation states that $C = \sqrt{gd}\,(1 + \frac{1}{2}H/d)$, while the last but one states that $C = \sqrt{gd}\,(1 + \frac{3}{4}H/d)$. In fact, the velocity distribution in a solitary wave is not uniform enough for the argument to apply; (see the paper *Proc. Camb. Phil. Soc.*, loc. cit.). There are many references in this book to internal reports some of which have now been published; the reader will be greatly helped if the references are revised up to date in the next edition.

Apart from these blemishes the book is well written, there are some good photographs, the drawings are adequate, and Mr. Russell's work may be regarded as a positive contribution to the subject.

Tides are much better understood than waves, and authoritative technical accounts exist, e.g. Doodson's *Admiralty Manual of Tides* to which frequent reference is made in the present work. Commander Macmillan has attempted a non-technical description of the theory, and as far as success is possible in such an undertaking he has succeeded. The last five chapters are on less conventional topics and are enlivened by the author's experience as Hydrographic Surveyor to the Southampton Harbour Board. There are a few misprints and trivial errors. Is Daniel Bernoulli correctly described as a French scientist on p. 182 ? The photographs are good, but the line drawings appear to have been reduced too much, some curves look spidery and the lettering cannot always be read with ease.

I think that the authors have succeded quite well in their task, and that this book will play a useful part in diffusing our knowledge of tides and waves. Dr. Herbert Chatley has written a foreword.

F. Ursell.

The skin of the earth. By A. Austin Miller. London (Methuen), 1953. Pp. ix, 198; 42 tables, 67 Figs. 21s.

Many will doubt the suitability of the analogy contained in the title of this volume but few will question the excellence of its contents. Based on some twenty-five years' teaching experience, this guide to practical work and introduction to research methods in Physical Geography contains over 100 exercises in the examination, analysis and correlation of geological, relief, meteorological, climatic and hydrological maps, charts and statistical data.

Part 1 – ' Topographical anatomy ' – is concerned with the geological and relief map. In 78 pages of lucid text, diagrams and sketches, the various problems met with in the construction and interpretation of these maps are explained. The preface's claim that the volume ' begins where most of them leave off ' seems strange reading, however, when one finds in this section explanations of simple terms like ' dip ' and ' strike ' and grammar-school exercises like the one on p. 44. Not the least surprising too is the statement (p. 49) that ' . . . . the *valley* is the cause of all topographic forms under " normal " erosion.'

Part 2 – ' The Circulatory systems ' – runs to 186 pages, and includes 23 illustrations. It is the one to which the meteorologist and climatologist will turn first and gives an interesting summary of the uses and limitations of the numerous methods for handling meteorological, climatic and hydrographic data. The trained meteorologist, after long practice and experience, is able to visualise and synthesize the miscellaneous distributions shown on the weather map. There is no short cut to the emulation of this mental process but others can arrive at quite reasonable and thorough appreciation of a particular situation, if more slowly, ' by pulling the station model to pieces and plotting separately those elements that are plottable.' In this

book the author encourages the student of geography to employ such objective methods in the scientific study of his subject. Exercises are given, and others are suggested, in the plotting of synoptic data, drawing various ispoleths, preparation and use of a simple geostrophic scale and simple temperature-height diagrams. The clarity of presentation of some of the more serious aspects of meteorology by means of simple temperature-height graphs on rectangular coordinates and arithmetical scales is to be commended, despite the inevitable ' sacrifice of precision and high-handed treatment of physical theory.'

Instructions and exercises are plainly given in a variety of graphical and cartographical devices for the analysis, arrangement and treatment of a wide range of climatic data, with rainfall and hydrology receiving treatment under such intriguing section headings as Ingestion (the analysis of precipitation), Metabolism (the function of water) and Excretion (the disposal of water).

Apart from one typographic slip on p. ix and the inserting of the solution of a geologist's field map between a consideration of river discharge and the regime of rivers, the layout, press work and reproduction of text and diagrams are of a high order. For students interested in acquiring evidence from maps and figures that is not immediately apparent from a superficial inspection, this well-produced volume is highly recommended.

G.M.H.

Publication of the Institute for Weather and Climate Research of the Norwegian Academy of Sciences and Letters, Oslo, 1953. No. 1; pp. 30, 1 Fig. N.p. No. 2; pp. 41, 2 Figs., 24 tables. N.p.

A new meteorological series, whose title is given above, has begun publication in Norway. Two numbers have so far been published. No. 1 on the two-dimensional perturbation of a flow with constant shear of a stratified fluid, by A. Eliassen, E. Høiland and E. Riis is a highly theoretical work. It deals with the perturbations of a non-viscous incompressible fluid, with density decreasing exponentially upwards, moving with constant shear in a non-rotating frame. The modes of oscillation of the system are found and it appears that the Richardson number is a fundamental parameter. The possible meteorological implications of this work are not clear.

The second number is a very elaborate statistical discussion by J. Nordø of a possible connection between solar activity and sea-level pressure. The general conclusion is that no convincing indication could be found of any change in the mean interdiurnal changes of pressure at sea-level over Norway after the occurrence of a strong solar flare. The work of B. and G. Duell and of others in this field is criticized.

G.A.B.

Emploi de la hauteur barométrique comme coordonnée verticale. By O. Godart. Brussels, Régie des voies aériennes, Service Météorologique. Bulletin Trimestriel. June/August 1953. Pp. 13; 3 Figs. N.p.

In this paper O. Godart, an originator with Dr. Sutcliffe of the use of pressure as a vertical coordinate, goes a step further to the use as vertical coordinate of the height corresponding, in a polytropic atmosphere, to a given pressure. His purpose in doing so is to find a coordinate which has most of the simplifying properties of the pressure coordinate in conjunction with the hydrostatic equation but which varies with actual height more steadily than pressure does. He sets up the equations of motion, continuity, vorticity and thermodynamics using a general polytrope, $p = A\,\rho^{n/n+1}$ but finally the I.C.A.N. standard atmosphere ($n = 4{\cdot}256$) is selected as standard polytrope.

The dynamical equations are not quite as simple as when pressure is used as the coordinate. The paper finishes with applications of the equations to the vertical variation of wind with height.

G.A.B.

German Climatological Atlases.

(a) *Klima-Atlas für das Gebiet der Deutschen Demokratischen Republik.* Berlin (Akademie Verlag), 1953. Pp. 19; 59 charts, 7 Figs. N.p.

(b) *Klima-Atlas von Hessen.* Bad Kissingen (Deutscher Wetterdienst), 1949-50. Pp. 20; 75 charts, 9 Figs. 22.50 DM.

(c) *Klima-Atlas von Bayern.* Bad Kissingen (Deutscher Wetterdienst), 1952. Pp. 23; 79 charts, 8 Figs. N.p.

Three large climatological atlases have been published in Germany during the past two years. Two, for Hesse and Bavaria respectively, were compiled by the meteorological service of the former U.S. zone of occupation and the third, for the Eastern zone of Germany, by the meteorological service of that region.

They are all very similar in content and arrangement; all have the main maps in colours and on the large scale of 1 : 1,000,000. They are mainly designed with agricultural requirements in mind and include some phenological as well as meteorological charts. The main source of information for all is the tables in *Klimakunde des Deutschen Reiches* (Reichsamt für Wetterdienst, Berlin 1939) and the period for most, but not all, quantities is 1881-1930.

Wind is the first element given in all and only mean directions are shown. Information on frequency of gales and intensity of highest gusts would have been useful for some purposes such as building design.

The mean temperature charts are of *wirkliche* temperature, which means that the values are not reduced to sea level. This unusual practice will make the maps for the mountainous areas difficult to use. There is much information on the periods of daily mean temperature below and above particular values, mean dates of first and last frosts and so on. Data of mean and extreme daily means and of earth temperatures are provided for one place in each region.

There follow maps of mean cloudiness, days of fog, sunshine, thunderstorms, mean rainfall and mean numbers of days of rainfall and snowfall. The extremes of monthly climate are shown by maps of the mean temperatures in a few individual exceptionally cold or warm months, very wet or dry months, or very snowy months.

An unusual quantity to be mapped is Reichel's dryness index which is, roughly, proportional to the ratio of mean annual rainfall to the mean annual temperature in °C plus 10°C. The exact significance of this index is not very clear. It is very strongly related to topography because of the rapid increase of mean rainfall with height.

There are no maps of exceptional days, e.g., distribution of rainfall on exceptionally wet days, such as are given in the *Climatological Atlas of the British Isles* which are clearly of great value in such work as drainage design. A map of stations would have been useful.

The phenological charts show mean dates of flowering of various plants and of the sowing and harvesting of major crops.

The production is very good indeed and the coloured plates which constitute the major part of the atlases are beautifully printed.

G.A.B.

Weather inference for beginners. By D. J. Holland. Cambridge University Press, 1953. Pp. xiv, 196; 50 maps and diagrams; tables. 30s.

There is sufficient of good in this book to show that the author could have written a much better book; as it is, it is inconsequential. Too much of the book deals with weather observations recorded by the author as a schoolboy in 1936; these observations are neither at one place for long periods, nor for a number of places at one time, but are a mixture at odd quarters of an hour extending from the City of London to Andover, Hants, to which station numbers have been allocated which bear no resemblance to the international system of numbering.

Some pages are devoted to cloud specifications which are a mixture of the descriptions and code numbers in use in 1936 and those now in use and it is difficult to assess the value of this to beginners for whom the book is written. Similarly, the worth of the diagram of a polar front on p. 4 is very much open to question.

The beginner's confusion will surely be added to when he reads the paragraphs on humidity on pp. 37 to 39; as an example we may quote :

' Its absolute humidity then, you see, is 2 per cent (of the air as a whole) while at saturation its *relative* humidity is 100 per cent (of this absolute amount). 2½ per cent, however, is rather rare, and with 3 you could hardly live.' And so on.

There are a number of clear weather maps on a blue and white background presumably referring to 1936, although this is not quite obvious; the description of these is discursive. There are also some pages on practical forecasting which make interesting reading but are more suited to a semi-scientific magazine.

The beginner in meteorology would be justified in doubting whether he had spent thirty shillings wisely.

R.M.P.

Atmospheric electricity. By B. F. J. Schonland. London (Methuen), 2nd Edition, 1953. Pp. vii, 95; 20 Figs, 8 tables. 7s. 6d.

A comparison of the present edition of this monograph with the first, which appeared in 1932, reveals that developments, during the intervening period, have been largely concerned with disturbed weather conditions. Many of the advances have been directly due to Dr. Schonland and his collaborators, and it is interesting to note that their major contribution – the use of the Boys camera to study lightning – is foreshadowed, in the last paragraph of the first edition, as a possible future line of research.

Of the four chapters in the original edition, two, upon the ionization and electric currents in the atmosphere, are retained with little change; the chapter on the penetrating or cosmic radiation is eliminated, this study having outstripped its parent; while the final chapter has been expanded in the second edition into three chapters, dealing with the electrification of thunderstorms, the lightning discharge, and charge generation in clouds, respectively.

There is little to dispute and much to admire in the monograph. Any defects are sins of omission rather than of commission, and these are understandable in a short work of this nature. Nevertheless, in view of the probable close connection between meteorological conditions and mechanisms of charge generation in thunderstorms, it is strange to find little reference to recent researches upon the meteorological structure

of thunderclouds. Another surprising omission is the absence of any account of modern work upon the charging of individual raindrops.

The monograph may be recommended with confidence to all students and more advanced workers in the field of atmospheric electricity, and, at the price, represents far better value than many more elaborate treatises.

E.T.P.

Fundamentals of thermometry and *Practical thermometry*. By J. A. Hall. Institute of Physics, 1953. Pp. 48 and 51. 5s. each.

These are among the first of a series of 'Monographs for Students.' They are intended primarily for 'students in courses for the Higher National Certificate in Applied Physics,' but will be read with interest by many others who like to know something of the background of the various aspects of their work.

They are, of course, superficial: in monographs of only 50 or so pages that is inevitable; but they are also somewhat loosely written and contain curious omissions and errors. For example: the accuracy of a mercury-in-steel thermograph certainly does depend upon the difference in level between the bulb and the recorder; the equation giving the relation between the resistance of platinum and its temperature, on the international scale, at temperatures below 0°C is a quartic not a cubic; and so on. Short bibliographies are provided.

R.F.

Annuaire de l'Observatoire Aerologique à Belgrade, 1951. Hydrometeorological Service R.P.F. Yugoslavia, 1953. Pp. ix, 68; tables, Figs. N.p.

This is the first number of the publication. It is in three parts: (*a*) pilot-balloon observations; (*b*) daily summary for 1951 of the weather over Yugoslavia; (*c*) a note on the cold-air outbreaks over Yugoslavia in 1951 by M. Čodež.

In the introduction it is stated that systematic radiosonde ascents are not as yet made in Yugoslavia but preparations for them are in hand. The pilot-balloon observations were made at 7 stations. At most one ascent a day at each station is published and the heights reached rarely exceed 4 km. The summary of weather gives for 17 well-distributed stations the types of weather for the day under 12 types together with information on the occurrence of low, medium and high clouds and the daily rainfall. The weather types are not specifically related to the prevailing air mass; they bear such titles as anticyclone-advective, advective-convective.

The third part contains small, not easily readable, charts of isochrones of cold-air outbreaks over Yugoslavia during 1951, with a brief note on each one. The outbreaks are numbered through the year according to the direction of arrival; 43 came from NW, 19 from N and 12 from SW. Surprisingly, none is noted from NE. Those outbreaks which were accompanied by a rise of pressure over Yugoslavia, to a higher value than had been noted earlier over neighbouring areas are specially noted and serially numbered.

G.A.B.

Values of some physical functions and constants used in meteorology. Definitions and specifications of water vapour in the atmosphere. Organisation Météorologique Internationale, Publication No. 79, 1951. Pp. 92, Sw. Fr. 2.

This booklet consists of two reports by the sub-commission on 'Physical functions and constants and aerological tables of the aerological commission.' One report is on the values of the main physical constants used in meteorology, mainly relating to the permanent gases of the atmosphere and their properties, and the second discusses the definitions and values of the physical properties of water in its various phases and of water-vapour air mixtures. Each report is followed by the text of a resolution adopted by the Conference of Directors at Washington in 1949, which summarizes the text and amplifies the tables of the sub-commission's report. This results in a certain amount of repetition, which, combined with the fact that the whole text is published in French and English on opposite pages, gives an air of ponderousness and spreads over 92 pages what could have been given in fifteen.

Though its use will be restricted, this booklet is to be highly recommended to those who teach meteorology (and to serious students) who should note the formal definitions of many quantities previously rather vaguely defined, and to those who do serious calculations on meteorology, for the numerical data. Both the precise definitions and the accurate data are very welcome. The present reviewer gives a special greeting to the tables of the vapour pressure over water and ice given at the end of the booklet. For the first time a consistent set of data is now available of the vapour pressure in millibars over the temperature ranges + 60°C to − 100°C and from − 150°F to + 140°F. The data for the vapour pressure over supercooled water go down to − 50°C (− 60°F). In both the Centigrade and Fahrenheit scale the vapour pressure is given for each whole degree.

A.W.B.

Faster than thought. A symposium on digital computing machines. Edited by B. V. Bowden. London (Pitman), 1953. Pp. xix, 416; 54 figs., 18 plates. 35s.

In Part 1 of this book Dr. Bowden gives an historical introduction to the subject of automatic computing and explains the main principles of the logical design of modern high-speed digital computing machines. He then goes on to describe briefly some of the circuit techniques now available and to explain how a typical machine is constructed and maintained and how it may be ' programmed.' Part 2 is devoted to brief accounts of almost all the computing machine projects now proceeding in this country. These are authoritative chapters mostly written by the actual designers of the machines concerned. Part 3 is also the work of several authors who provide descriptions of many applications which amply illustrate the versatility of such machines. In one of these chapters Dr. R. S. Scorer discusses the role which high-speed computers may play in meteorology.

The historical account gives just prominence to the ideas of Charles Babbage who may be said to have founded the subject over a century ago. Considerable mention is also made of the part played by Lady Lovelace, only daughter of Lord Byron, who understood, perhaps more fully than his other contemporaries, the potentialities of the ' engine ' which Babbage designed and began to build. She translated into English an important publication on this engine by an Italian engineer, L. F. Menebrea. This translation, together with a penetrating commentary by the translator herself, was published in Taylor's *Scientific Memoirs* in 1843. A very welcome inclusion in Dr. Bowden's book is an appendix reproducing in its entirety this interesting document which has been out of print for much too long.

This volume is a really valuable addition to the literature on automatic computing. It can be strongly recommended to the newcomer to the subject as a very readable – and entertaining – general account and a useful source of collected information about contemporary British machines. It is not a text-book, although those who wish to explore the finer detail will find suitable references at the end of most chapters. The reader should be warned, however, that the sparse details of American work on this rapidly progressing subject are far from comprehensive and no longer up-to-date.

Dr. Bowden's easy and witty style makes fascinating reading of his personal contributions but in Parts 2 and 3 the symposium nature of the book becomes apparent and there is inevitably some lack of uniformity since editorial intervention has, rightly, been kept to a minimum. It would have been helpful, however, if a statement of the authorship of each chapter had been contained in the chapter itself. As it is, the list of contributors contains an error : Dr. M. V. Wilkes was responsible for Chap. 7 *not* Chap. 17.

E. N. Mutch.

Le Pioggie della Libia. By A. Fantoli. Ministero dell' Africa Italiana, Ispettorato Meteorologico, Roma, 1952. Pp. 529; many figures and tables. N.p.

Dr. Fantoli has satisfied a great need by making available a vast amount of rainfall data for this large and important area. Previously published data were very limited and fragmentary.

In an introduction the author divides the area into seven geographic regions, and he specifies six rainfall types.

He then gives a concise account of the air masses and frontal systems which are most important for the production of rain in Libya, and discusses the most important synoptic weather types which produce rain. 90 to 95 per cent of the rain falls during the period October to March, mostly at cold fronts or in post-frontal showers, with north-westerly winds.

In chapters two and three there are detailed discussions of rainfall data for the chief cities of Libya. That for Tripoli is very complete, and most valuable. For the period 1879 to 1948, with few gaps, the rainfall is given for each ten-day period; there are detailed monthly frequency distributions of days with falls of particular amounts; the heaviest falls in each month of amount greater than 20 mm; lists of all periods with total rainfall greater than 20 mm with each day's fall recorded; lists of all periods with rain on three or more consecutive days, with the rainfall amounts for each day; and lists of periods of droughts lasting 15 or more consecutive days. In addition there is a statistical analysis of the annual rainfall of Tripoli, including the fitting of a logarithmic-normal distribution to the annual falls. Shorter discussions follow for the other major stations.

In chapters four and five there is a detailed analysis of the geographical distribution of annual rainfall followed by a discussion of rainfall in connection with agriculture. The 200 mm isohyet, which encloses only about 2 per cent of the whole region, marks the limit of considerable natural vegetation and also of profitable cultivation. Even in areas with average falls above 200 mm the climatic hazards in cultivation are considerable. Rainfall may be scarce for three consecutive years; and on occasions much of the rainfall may occur in a few heavy falls, doing perhaps more harm than good.

There are tables for more than 200 places giving the total monthly rainfall and number of days of rain for each year of available records. The length of record is mostly between 7 and 15 years, but about 25 stations have records of over 20 years; and Benghasi and Tripoli have 50 and 70 years respectively.

There are many diagrams to illustrate in particular the spatial variation of annual rainfall, both locally and over wide areas; and two large pocket maps with annual isohyets for western and eastern Libya.

Dr. Fantoli deserves thanks and congratulations for his work. It is to be hoped that he will find time to make analyses for this area of other meteorological elements.

A. F. Jenkinson.

The Antarctic today. New Zealand Antarctic Society, Editor F. A. Simpson. Wellington, N.Z. (A. H. and A. W. Reed), 1952. Pp. 389; 45 plates; 50 figs., 2 tables, one folding map in back cover. 47s. 6d.

Antarctica and the wild seas about it loom larger in the interest of the nations, since it has been realized that territorial claims can only be substantiated by effective occupation. The days of transient expeditions are being overtaken by the establishment of permanent stations, a departure in which British Commonwealth countries have so far taken the lead. Meteorologists too have an interest for more particular reasons, and some are taking a prominent part in activities in the Far South.

New Zealand has been repeatedly used as a base for expeditions to the Antarctic since the days of Captain Cook (1772-4). From 1860 until 1929 the New Zealand government maintained food depots for wreck survivors on sub-Antarctic islands and sent an annual rescue patrol. Since 1941 a weather station has been run on Campbell Island (52½°S, 169°E); but little has yet been made of the Ross Dependency (south of 60°S, between 150°W and 160°E), granted to New Zealand in 1923. The appearance of this book is a welcome sign of reawakening concern, and congratulations are due to the professional people and interested laymen, associated in the New Zealand Antarctic Society, which has produced it.

This is a pleasantly illustrated book, containing both articles to stir the sense of adventure and serious summaries of various fields of exploration and research. All the great men of exploration and the institutions which supported them are recorded, and the book is up to date as regards plans for the future.

There are well-informed chapters on Antarctic geology (cautious yet remarkably comprehensive and well referenced, and incidentally summarizing broadly the evidence on climatic changes); on the Southern Ocean and its water circulations; on marine biology; on whaling and sealing and bird-life and so on. The chapter on meteorology is good; differences between northern and southern hemisphere patterns of frontogenetic zones are stressed, but the significant contrast in frequency of middle-latitudes blocking anticyclones might be more clearly stated. The assertion that the southern polar front is formed by advances of the Antarctic front should not exclude Palmer's suggestion of rejuvenation of old occlusions, which are frequently transformed into the main cold front. There is a misleading statement elsewhere on p. 115 that low salinity in Antarctic surface water is partly due to high precipitation (believed to be 4-12 inches per annum in this zone). There are useful chapters on aurora and ionosphere research, though the latter hardly touches the Antarctic as yet.

'Antarctic' and 'sub-Antarctic' zones are best defined in terms of the antarctic convergence and of the coldest and intermediate surface water masses of the Southern Ocean (as on p. 129) rather than the excessively broad usage elsewhere in the book (suggested on p. 8). There is also a strange usage of "glacierization' for glaciation in Chapter II.

Some overlapping and contradictions may be unavoidable in a book contributed to by authors as far apart as S. Africa, Australia and New Zealand. An editorial disclaimer is printed on p. 4. One notices differences as to whether there was any Antarctic glaciation before the Quaternary period and as to morainic evidence (necessarily mostly on the continental shelf) of ice erosion.

More serious criticisms are the lack of any heights of land features, both on the 1 : 16 m map and in several of the descriptive chapters, and misleading and curiously final statements in the first two chapters (pp. 17 and 28, the latter contradicted on p. 364) regarding the greatest known elevations of ice-cap and mountain features – both almost certainly put in the wrong part of the continent. Mount Siple at the Pacific Coast in 74°S, 124°W, discovered about 1940, is believed to exceed 20,000 ft; and in the Atlantic sector, near 75°S, 0°W, the inland ice was declared by a German expedition in 1939 to reach 15,000 ft, though some doubt has since been cast on the reliability of this figure.

The geography of the unexplored interior of East Antarctica is still a field for conjecture, about which meteorological evidence and reasoning has so far made the most specific suggestions. There is ample room for continued efforts both by the interested northern peoples and the younger nations of the southern hemisphere in finally unlocking the secrets of this vast terrain.

H.H.L.

Thunderstorm Electricity. Edited by Horace R. Byers. Chicago (University Press), 1953. Pp. viii, 344; Figs., tables. $6.

Readers who are actively interested in this subject will probably recall that a conference on it, sponsored by the U.S. Air Force Geophysics Research Directorate, was held at the University of Chicago in April 1950. The report on this conference, which had a limited circulation, aroused sufficient interest to warrant making its contents more widely available in the book now under review.

This book, however, is not merely a reprint of the report. Most of the contributions have been revised and brought up-to-date. Also, some chapters by authors who did not take part in the conference have been added. These changes have not resulted in a text-book on thunderstorm electricity and the publication remains very much a collection of specialist articles on the various aspects of the subject. The quality of the articles varies considerably, though not as much as their length, which runs from seven pages for the shortest to ten times as many for the longest. One or two of the articles would be more appropriate as papers in a scientific journal; others are more general surveys. Space does not permit discussion of all the sixteen chapters but the following remarks should indicate the scope of the book.

After a brief introduction by the editor there is an interesting account in Chapter II by H. Israel of the atmospheric electric field and its variations. This is confined mainly to the phenomena of undisturbed conditions, but it should be useful in giving the non-specialist reader the basic ideas on which the explanations of thunderstorm electricity must be founded. In this respect Chapter IV by Byers and Braham on thunderstorm dynamics is also useful; it is based largely on their well-known report *The Thunderstorm* of 1949. Chapter III by L. Konigsfeld is one that would be more appropriate in a scientific journal. It describes electronic apparatus for measuring the earth's electric field by radiosonde and gives some illustrations of the variation of the field with height. As, however, most of these refer to measurements during a solar eclipse they are not very relevant to the subject of the book. The only example given of a sounding through a cumulonimbus cloud is described as showing very large potential gradients above and below the cloud, though they did not appear to exceed 200 v/m, which is not much more than the average fine-weather field.

By far the longest article in the book is on the formation of precipitation in cumulonimbus, by H. Weickmann. It contains a lot of useful information on the microphysics of convection clouds but in comparison with the rest of the book the account is, in parts, very detailed; for example, hailstones are given very comprehensive treatment. In Chapter VI Workman and Reynolds describe their experiments of 1950 on the generation of charge by the freezing of water and on the effect of impurities, but they do not appear to have modified their account in the light of more recent research. A very interesting survey is given in the next chapter by L. B. Loeb of researches on the processes of static electrification and their part in thunderstorms; this chapter also has a wider application to static electricity hazards generally. In the next chapter Ross Gunn summarizes his published work on measuring the electricity caused by precipitation particles. Investigations by Seville Chapman described in Chapter IX suggest that charge generation by the disruption of drops cannot be ruled out as an important thunderstorm process.

The last six chapters of the book are concerned with measurements of the effects of lightning. Malan and Schonland review their work on charge distribution deduced from lightning measurements and give evidence supporting the view that the sudden freezing of supercooled water droplets is one of the main processes, and possibly the original triggering process in the electrification of thunderstorms. Pierce and Wormell, in Chapter XII, describe results of studies of field changes and wave forms of atmospherics and conclude that there is no early prospect of replacing the multi-station direction finding method by the use of wave forms for locating thunderstorms. In Chapter XIV, the second longest in the book, H. Norinder gives an interesting historical account of the development of methods of locating thunderstorms. It seems that the Russians can justly claim to have been first in this field, for their physicist Popoff devised a receiver and recorder of atmospheric disturbances nearly 60 years ago. Norinder describes modern sferics techniques in some detail and in addition to the use of the direction-finding method of location for forecasting purposes he advocates the wider use of recording goniometers for the statistical investigation of the distribution and duration of thunderstorms throughout the world. Confirmation of existing estimates by reliable recording methods is highly desirable. The last two chapters of the book are brief notes on the effect of lightning on aircraft and on the protection of power lines.

The reviewer found at least three-quarters of the book interesting and useful. It provides a good source of information on recent researches on thunderstorm electricity. It is a pity it was not rounded off with a general discussion of the various theories of charge generation.

The text appears to have been reproduced by a photographic process and it is very well done. The book is plentifully supplied with clear illustrations but unfortunately there is no index and only a list of chapter titles is provided for finding one's way about the book.

F.J.S.

Obituary Notices

Dr. L. F. RICHARDSON, F.R.S.

Lewis F. Richardson, who died at his home at Kilmun, Argyllshire, on 30 September 1953, was born in 1881 at Newcastle-upon-Tyne. He was educated at Bootham School, York, the Durham College of Science, Newcastle, and King's College, Cambridge, graduating at Cambridge with First Class Honours in the Natural Sciences Tripos in 1903.

I first met Richardson when I entered the University College of Wales, Aberystwyth, as a student in 1904. He was then a demonstrator in the Department of Physics, and at that time he was keenly interested in designing a planimeter. He carried his model in his pocket, and would take it out and test and modify it in any free moment. He remained in my memory as a quiet friendly man, always ready to help in any difficulty with an experiment. I think he must have left Aberystwyth soon after my arrival to take up a post on the staff of the National Physical Laboratory, where he remained for three years. He then worked for three years in a tungsten electric-lamp factory.

In 1913 he was appointed Superintendent of Eskdalemuir Observatory. He had in 1910 published his first paper, which appeared in the *Philosophical Transactions of the Royal Society*, on the application of the method of finite differences to the solution of the differential equations of physical problems. He now conceived the idea of applying this method to atmospheric problems, with a view to applications to weather forecasting. He wrote a first draft of the method of ' weather prediction by numerical process,' which Sir Napier Shaw communicated to the Royal Society for publication, in 1916. As the paper was considered too long for publication in the *Philosophical Transactions* the Royal Society offered to contribute to the cost of publishing it in the form of a book, and it was finally published in 1922 by the Cambridge University Press.

Richardson remained at Eskdalemuir until 1916, when he joined the Friends' Ambulance Unit, and was drafted to France, where he continued to serve until the end of the war. As a Quaker, Richardson was a conscientious objector, but he served gallantly and uncomplainingly through times of danger and strenuous effort.

On his return to England after the end of the war, he was sent to Benson, to assist Mr. W. H. Dines. There he found ample scope for his ingenuity. He devised a method of measuring winds at distances of hundreds of feet above the ground, by shooting spheres upward. In his hands this became a practical method of observation, the projectile being shot vertically upward with a rifle, and its return to earth being noted by the observer, who measured the magnitude and direction of the wind shift. Richardson was very happy in his work at Benson, but when in 1920 the Meteorological Office was taken over by the Air Ministry he resigned his post, as he refused to be associated with the fighting forces in any capacity.

Richardson was appointed Lecturer in charge of the Physics Department at Westminster Training College in 1920, and remained there until 1929, when he was appointed Principal of Paisley Technical College, a post which he held until his retirement in 1940. At Westminster College he found time to carry on his researches in meteorology. He completed the preparation of his book on *Weather Prediction by Numerical Process*, which he had considerably revised while on active service in France, where his manuscript was twice lost and again found, the second time under a heap of coal.

Richardson became a Fellow of the Royal Meteorological Society in 1919, and served as Secretary from 1921 to 1924. He was elected F.R.S. in 1926, in which year he also obtained the degree of D.Sc. (London). His interests swung over about this time from meteorology to the mathematical and psychological analysis of the causes of war. He obtained by examination the degree of B.Sc. (London) in Psychology in 1929, and it is notable that during the years 1931 to 1947 he published no papers on meteorology. He devoted his available time during these years to the analysis of the causes of war. A book giving some of the results of his studies, entitled *Generalised Foreign Politics* appeared in 1939, but a revised and extended edition entitled *Arms and Insecurity* remains unpublished, except on microfilm, though parts of it have been published in various journals. His thesis was that war inevitably arises from warlike preparations, and later generations may prove more receptive to his ideas than was his own generation.

Richardson was completely sincere and consistent in his detestation of war and was profoundly shocked when in the inter-war years he found that those interested in the diffusion of war gases were most interested in his meteorological researches. So shocked was he, that he proceeded to destroy all the unpublished papers he had still in his possession and until 1948 he published nothing meteorological.

Richardson's researches in meteorology covered a wide field. A bibliography of his papers in meteorology, compiled by himself, together with brief comments on some of the papers and corrections to others, appeared in *Weather*, January 1949. He published a joint paper with H. Stommel in the *Journal of Meteorology*, **5**, p. 238, 1948, and his last paper, on the eddy-diffusion of clusters, in *Proc. Roy. Soc.* A, **214**, p. 1, 1952.

His meteorological contributions fall into two distinct fields. In the first of these, the study of atmospheric turbulence, Richardson will be best known by what is now called the ' Richardson number,' which determines whether turbulence will increase or decrease. This number is in the category which includes the Reynolds number and the Mach number. His book *Weather Prediction by Numerical Process* was perhaps 30 years too early. Richardson showed that it would take 60,000 computers, working at high speed, to compute tomorrow's weather charts before tomorrow arrived. At that time the facilities offered by the modern electronic high-speed computer were not available, while the available observational material, especially in the upper air, was insufficient to give a complete picture of the state of the atmosphere. To-day's network of surface observations, and our much improved methods of measuring winds in the upper air, offer a much

more complete picture of the state of the atmosphere at any given time, while the modern electronic computing machine can carry out in relatively few minutes computations which by ordinary methods would require thousands of computers over many months.

Thus, the work of Richardson on the use of numerical methods in predicting tomorrow's weather is now to be regarded as likely to be fruitful in the near future.

As a man, Richardson was patient and kindly, steadfast and courageous. His pupils speak of him as a clear teacher, with his own original ways of illustrating the laws of physics, and with a kindly interest in each of his students. He leaves behind many, both among his former students, and among the scientists who knew him, who will remember him as a shining example of moral strength, fortitude and kindliness.

D.B.

Mr. J. FAIRGRIEVE, M.A., F.R.G.S.

James Fairgrieve, a Fellow of the Society since 15 May 1918, a former Vice-President (1932-33) and for many years one of the most familiar figures at the monthly meetings, died in Barnet General Hospital, Hertfordshire, on 8 October 1953 after a prolonged illness. Eldest son of the Reverend George Fairgrieve, he was born in Ayrshire on 14 March 1870 and received his early education at Saltcoats, Ardrossan, and Glasgow High School. He then entered University College, Aberystwyth as an exhibitioner and later, having taken an external degree at London University, proceeded to Jesus College, Oxford (where also he won an exhibition) and graduated with honours in mathematics. It was as a mathematical master that Fairgrieve's long and successful teaching career began – at Kelso and Campbeltown. But various influences, notably his association with Herbertson at Oxford, with H. J. Fleure, and with Halford Mackinder, under whom he studied at the London School of Economics, led him when still a young man to the conviction that his special abilities could better be devoted to the overdue reform of instructional methods in geography. In 1899, soon after his marriage with Emily Croft, of Bradford, he and his wife came to London and opened the New Southgate High School. Here he put his schemes into experimental practice. Eight years later he gave up his headship to become geography master and second master at the William Ellis School, Highgate. While there he broke new ground by promoting geography to an equal place in the curriculum with physics and chemistry. In 1912 he accepted a lectureship (subsequently to become a readership) in education with special reference to geography at the old London Day Training College for Teachers – afterwards the Institute of Education of the University of London. In that capacity he exerted for nearly a quarter of a century, until his retirement under the age limit in 1935, a profound influence on successive generations of trainees seeking their teacher's diploma after graduating at the University. From 1927 to 1935 he was head of the Department of Colonial Education. He also served as an original member of the Commission on Cultural and Educational Films, and in 1935 was president of the Geographical Association. Few men can have played so important a part as he did in raising geography from its Cinderella-like position in tuitionary schemes half a century ago to the high status which it now enjoys among the major subjects of humanist teaching in our schools and universities.

Fairgrieve was a prolific writer, blessed with a simple yet forceful style, a lively sense of humour and strong powers of imagination. It has been said that he was something of a prophet. One of his early books, *Geography and World Power*, though dating from before the 1914-18 war, is still in print and pertinent to the international situation, all subsequent upheavals having necessitated but little revision of the text. Most of his later works were written in collaboration with Ernest Young. These included the well-known series *Human Geographies*, comprising six books designed for primary and three for secondary schools, all of which have passed through numerous editions. Another series, entitled *Real Geography*, was brought to completion only a year or so ago, when Fairgrieve was an octogenarian in seriously failing health. His contributions to meteorological literature, while not very extensive, were of substantial value. A pioneer investigation was his extremely detailed study of the development and travel of the rain-fields associated with the great 'Derby Day' thunderstorm on 31 May 1911 (see Vol. 38 of this *Journal*, pp. 105-116 and *British Rainfall* 1911, pp. 33-39). This was based on about 14,000 plottings of information supplied by the staff and 300 boys of the William Ellis School and some 400 other observers, official and casual, in London and the adjacent counties.

Fairgrieve was one of the kindliest of men, a good companion immensely popular among a host of friends and former colleagues and pupils; his genial presence, tall lank form, tangle of iron-grey hair, walrus moustache and perpetual overcoat will be much missed wherever either geographers or meteorologists gather together in London. At a memorial service for Mr. Fairgrieve held at Christ Church, Woburn Square, London on 21 October 1953 the Society was represented by Mr. Ernest Gold, C.B., D.S.O., F.R.S.

E.L.H.

Mr. J. ASPINALL

By the death of James Aspinall on 17 October 1952 at the age of 72 the Society lost a Life Fellow of 47 years' standing. Mr. Aspinall, who latterly resided at Altrincham, Cheshire, was chairman and managing director of Messrs. Wm. Bailey & Co., Ltd., the Manchester firm of sanitary and heating engineers, with whom he had been actively associated for well over half a century.

E.L.H.

Mr. L. STARBUCK

Leonard Starbuck, who had been a Fellow of the Society since 1938, died in a London hospital on 15 September 1953 after a long illness. He was only 42. After obtaining his degree in physics at Nottingham University, he worked as a scientist at Porton until 1936, when he joined the staff of the Royal Observatory, Hong Kong. The Colony which became his home for the rest of his life offered many opportunities to a man of his energy and talents. His professional work included forecasting, general meteorology, terrestrial magnetism and seismology, and in his spare time he took part in many useful activities.

He was captured on the first day of the Japanese war while dismantling a magnetic station near the frontier. During the years of his captivity, he maintained the medical statistics of the prison camp; his ciphered notes formed the basis of a valuable report on the incidence of disease and malnutrition among the prisoners.

On his return to duty he rendered most valuable service during the restoration and re-organization of the Observatory. He was promoted Assistant Director in 1947, and on more than one occasion subsequently acted as Director. Always keenly interested in international meteorology, he represented Hong Kong at the I.M.O. conferences in Toronto and Washington in 1947, and was a member of the W.M.O. Commissions for Aeronautical and Maritime Meteorology. He was also a Fellow of the Institute of Physics.

In spite of his many routine duties, he found time to carry out some statistical investigations on tropical cyclones and climatology, which were published by the Observatory. He was a gifted speaker and writer, and did much to popularize meteorology in the Colony. His loss will be keenly felt not only by a wide circle of friends in Hong Kong but by many meteorologists from other countries with whom he came in contact.

G.S.P.H.

Mr. P. E. NEGRETTI

Paul E. Negretti, Chairman and Managing Director of Negretti and Zambra, Ltd., died on 20 September 1953, at the age of 70. He had been associated with this well-known firm of makers of meteorological and other scientific instruments since the early years of the century, and guided its development and policy over a long period which included the difficult war years.

T.W.W.

Corrigenda to July 1952 issue

Our attention has been drawn to the following corrigenda in the paper by S. M. A. Haque, 'The initiation of cyclonic circulation in a vertically unstable stagnant air mass' (p. 394):

p. 394. Eq. (1), $\boldsymbol{\omega}$ is a vector quantity and should be in bold type in the equation and in the next line.

p. 395. Eq. 1 (a), the second term should be $-\frac{V_\varpi^2}{r}$.

Eq. 1 (b) should read $\frac{dV_\varpi}{dt} + \frac{V_r V_\varpi}{r} + \ldots\ldots$.

Fifth line from the bottom, should read: The Eqs. 1 (a), 1 (b), 1 (c), $\ldots\ldots$.

p. 396. Eq. (6) should read $\left(\frac{\partial}{\partial z} - \frac{\partial S}{\partial z}\right)(\ldots\ldots) - \left(\frac{\partial}{\partial r} - \frac{\partial S}{\partial r}\right)\left\{\frac{dV_z}{dt} + m(V_\varpi \sin\varpi - V_r \cos\varpi) + g\right\} = 0.$

p. 397. Eq. 9 (b) should read $V_\varpi = 0 + v_\varpi$.

Eq. (13) should read $\frac{\partial s}{\partial t} + Bv_z = 0$.

Eq. (14) should read $\ldots\ldots + \frac{\partial v_z}{\partial z} = 0$.

p. 399. Penultimate line should read $gB + q^2 < 0$.

p. 400. Lines 5, 7, 9 and the bottom line, Z_0 is to be replaced by z_0.

Line 6, z is to be replaced by z_0.

Bottom line, Z is to be replaced by z.

p. 401. Eqs. (29) and (30), r is to be replaced by r_0.

p. 402. Lines 6 and 10, Z_0 is to be replaced by z_0.

p. 405. Line 17, Z_0 is to be replaced by z_0.

p. 406. References; the second reference should be to Milne-Thomson.

Corrigendum to October 1953 issue

Discussions

Gordon Manley, p. 563, Table 1—

Column 2, 1736 winter, *for* 44·3 *read* 40·9.

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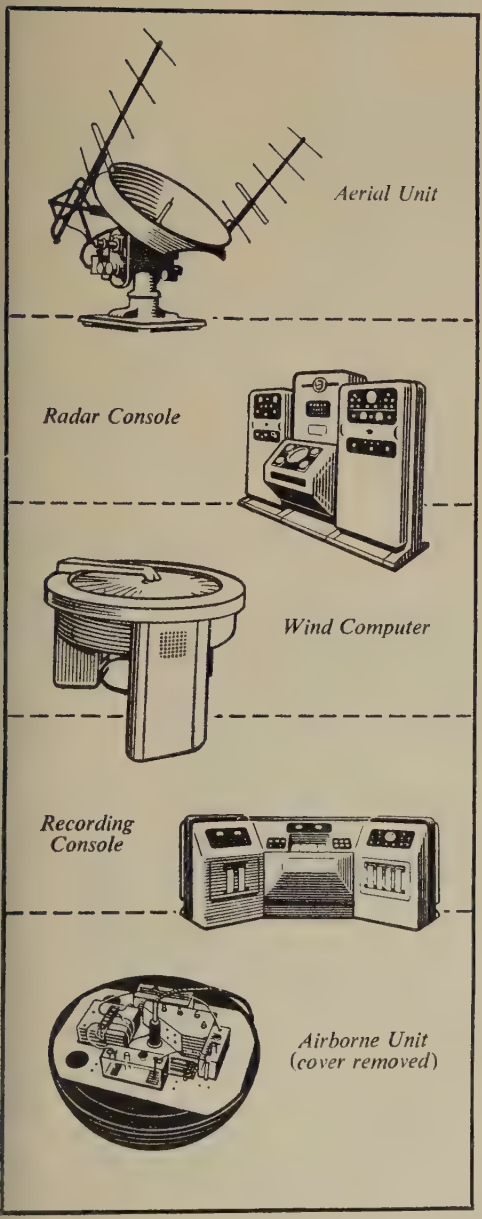

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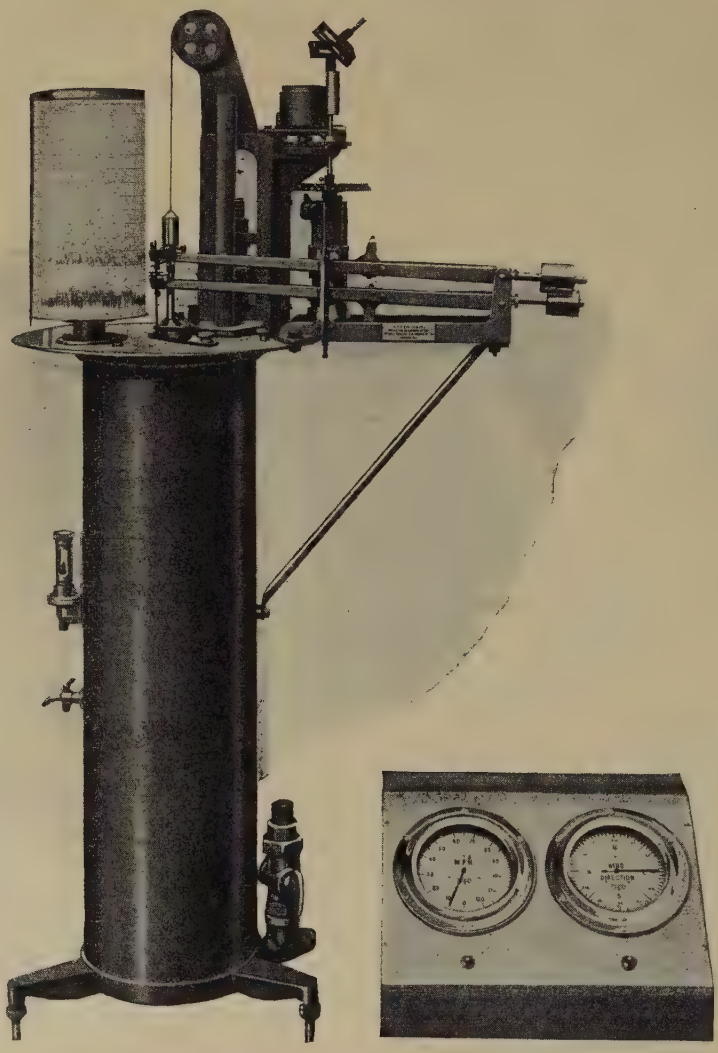

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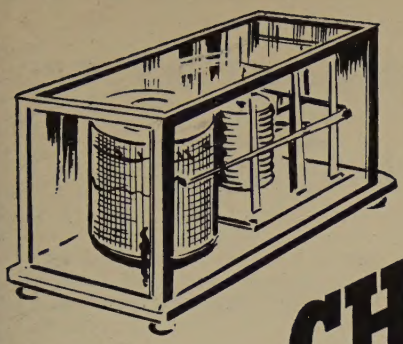

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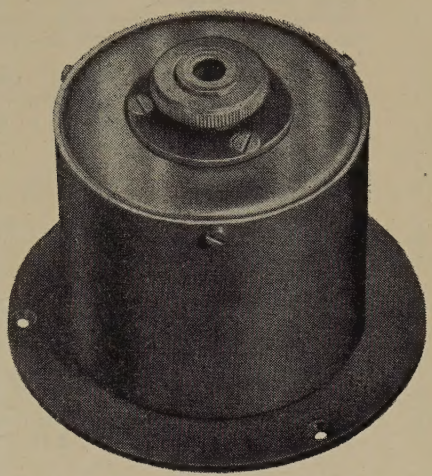

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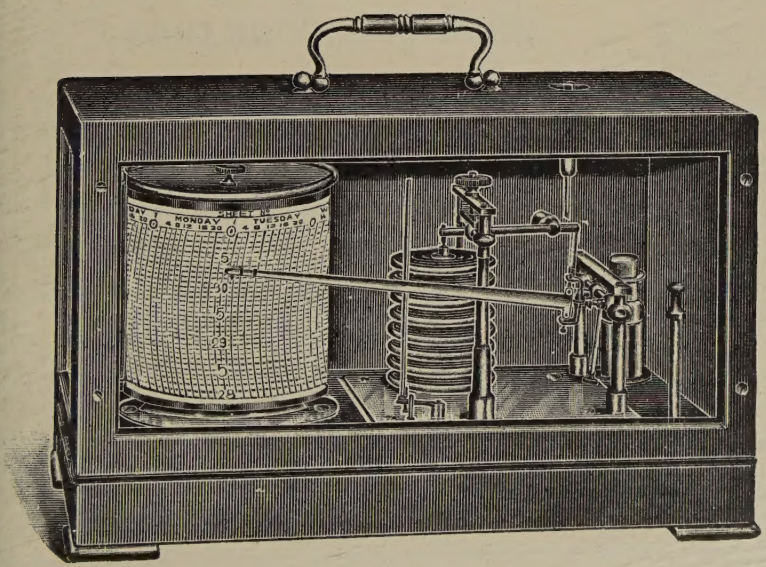

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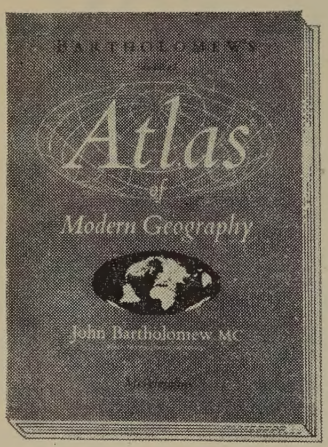

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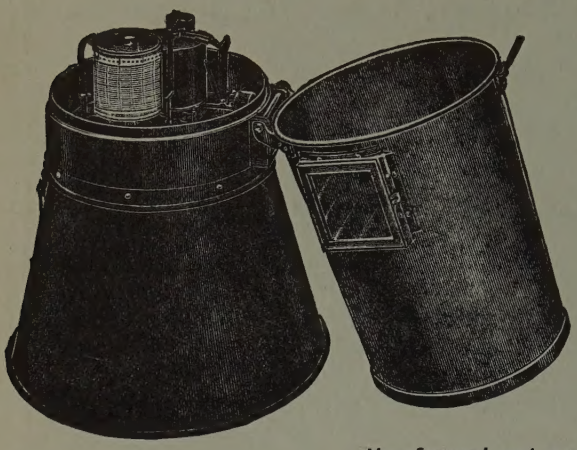

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